Cartoon vignettes for connecting mathematics education theory and practice –

Supporting teacher education and professional development through collaborative reflection on mathematics classrooms

--- Preprint ---

Sebastian Kuntze, Ceneida Fernández, Marita Friesen, Pere Ivars, Jens Krummenauer, Salvador Llinares, Libuše Samková, Karen Skilling, Lulu Healy, Melania Bernabeu, Felix Schwaderer

Co-funded by the Erasmus+ Programme of the European Union

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0. How to read this book

This book might be relevant for readers with different backgrounds. For this reason, we would like to indicate possible ways of reading this book. If you aim at …

- **… Promoting pre-service teachers' criteria-based analysis of practice contexts:** In this case, the introduction (section 1) into relevant theoretical background elements could be a starting point for then finding out more about vignettes, their design for profession-related learning, examples of vignettes and the framing of vignette-based work in sections 2, 4, 9 and 11, in particular. The activities following up on these sections might help you to deepen your understanding of the corresponding sections and to think about your own possibilities of promoting pre-service teachers' criteria-based analysis of practice contexts. If you would like to extend this interest towards vignette-based research into pre-service teachers' analysis and/or their growth with this respect, then section 5 might also be of interest for you.

- **… Stimulating collaborative reflection on classroom practice with in-service teachers:** Under this perspective, sections 2, 4, 7, 9, and 11 might be of particular interest. Even if some of the examples report on vignette use or course concepts for the teacher education context, adaptations for in-service teachers may be carried out easily, as vignette-based work in in-service professional development activities can - in many cases - directly profit from developments for pre-service teachers. The related activities can help you to adapt the content of the above-mentioned sections to your personal goals in in-service mathematics teacher professional development. The theoretical background section 1 may provide you with frameworks for describing the learning goals in the domains of teachers' professional knowledge, noticing and/or aspects of the competence of analysing, related awareness, teachers' views, etc.

- **… Researching teachers' growth in noticing / analysing classroom situations:** For a rather research-oriented perspective, we recommend especially section 1 for an introduction to some aspects of background theory, and sections 6 and 11 for thoughts about vignette-based research designs. Further, you might be interested in the research

publications which have emerged from the coReflect@maths project work. A list of all project-related publications can be found on coreflect.eu, some of these publications are also included in this booklet's references list.
As vignette-based research is a dynamically growing field with steady methodological innovations, the ideas related to profession-related learning and reflection in the further sections of this book might also be of interest for you, as specific forms of vignette use could be transferred to a use in research frameworks.

We hope your reading of this booklet is fruitful for your work - the coReflect@maths project team is interested in your vignette-based work as well – do not hesitate to contact us if you feel that you would like to follow up on the booklets' contents.

1. Introduction

Promoting reflection on classroom situations can be helpful for linking theory and practice and related professional growth of pre-service teachers. This book aims at showcasing examples and providing useful information for designing and evaluating professional learning opportunities around such classroom situations. By representing classroom situations in so-called vignettes, they can repeatedly be used as opportunities for reflection, in professional learning settings for pre-service mathematics teachers or in-service teachers who are experienced in the mathematics classroom, as well as for research purposes.
In the following, we will outline relevant theoretical frameworks which can be used for describing vignette-based work in such professional learning contexts and its potential effects.

1.1 Approaches to elements of mathematics teacher expertise

A key feature of mathematics teacher expertise is the optimal support of the students' mathematics-related learning. For optimally supporting students, professional knowledge (e.g. Shulman, 1986; Ball, Thames & Phelps, 2008, Kuntze, 2012) is considered as a prerequisite. Potentially inert professional knowledge, however, is not enough: Kersting and colleagues (2012) point to the aspect of the "usability" of such knowledge in classroom situations, i.e. the accessibility to components of professional knowledge when having to analyse processes or events in the classroom. In a somewhat analogous way, the concept of teachers' noticing (e.g. Sherin, Jacobs & Philipp, 2011) highlights that mathematics teachers have to be able to reason on classroom situations in a knowledge-based way – they hence need the competence to draw on their professional knowledge when observing and interpreting processes in the classroom. Moreover, individual dispositions may be relevant for teacher expertise, such as teachers' views (e.g. Törner, 2002) or awareness (Mason, 2002). Even if there may be good reasons for theory-driven approaches to describing aspects of mathematics teacher expertise, relevant aspects of mathematics teacher expertise basically have to respond to the requirements of practice of the teaching profession and hence may get salient related to classroom situations or other practice-related contexts. For describing aspects of mathematics

teacher expertise, questions such as “What do experts know?”, “How do they think?”, “How do they perceive situational contexts of their profession?”, “How do they use their knowledge?”, or “What convictions or views do expert teachers hold?” are in the focus. Moreover, rather normative questions such as “What should teachers know or be able to do for theoretical reasons?” can play key roles. These questions can apply to a variety of professional requirement contexts: Expertise can become salient when mathematics teachers explore what learners think, when they analyse or select tasks, when they act or react in the classroom, when they reflect on instructional practice or when notice crucial events in interaction processes in the classroom. Situation contexts thus are meaningful points of reference for research approaches to mathematics teacher expertise – and for theory development in this area: Several approaches have taken such a perspective on situated requirements related to the profession of mathematics teachers: For example, notions such as “professional vision” (Sherin & van Es, 2009), “usable knowledge” (Kersting et al., 2012), “noticing” in the sense of “selective attention” (cf. e.g. Seidel et al., 2013) or in the sense of “knowledge-based reasoning” (Sherin, 2007, Seidel et al., 2013), as well as the notion of “awareness” (Mason, 2002) describe aspects of mathematics teacher expertise which are directly connected to classroom situations. These respective constructs however are not understood as limited to a set of specific classroom situations, but considered in more general ways, enlarging the perspective beyond the specific situation context(s) used in the respective research designs. This points to substantial challenges, not only on the level of research methods, but also on the level of the design of related learning opportunities: bridging the gap between the specific classroom situation(s) and the non-situation-specific, general meaning of the construct (related to an aspect of teacher expertise) needs careful consideration, e.g. related to validity.

In contrast to the approaches mentioned above, there are models of professional knowledge which aim at giving a relatively complete panorama of areas of mathematics teacher expertise: The constructs presented in the models by Ball, Thames & Phelps (2008) or by Baumert & Kunter (2006, p. 482), for instance, are intended to cover teacher expertise in an overview-like way. The professional knowledge components and further dispositions in these approaches however appear as relatively distant to specific classroom situations. Leinhardt & Greeno (1986), however, assume that professional knowledge is often structured in an episodic way. For this reason and complementing the above-mentioned models, in the model of components of professional knowledge used in Kuntze (2012), the aspect of

situatedness of professional knowledge components is acknowledged: This model distinguishes more general components of professional knowledge (which are rather not specific for particular classroom situations or content) on the one hand from components of professional knowledge which are closely connected to content domains, to specific content or even to specific classroom situations on the other hand. This model may facilitate the description of aspects of teacher expertise related to professional knowledge in the full range of situatedness, from specific classroom situations or content to more general knowledge domains, e.g. views related to mathematics in general or general knowledge about mathematics teaching and learning.

1.2 An example of a theoretical approach to teacher expertise: criterion-based noticing, analysing, awareness

Still, for all components of professional knowledge, being able to draw on knowledge-based criteria when faced with professional requirement contexts in practice is key (Kersting et al., 2012). “Noticing” in the sense of “selective attention and knowledge-based reasoning” (Sherin et al., 2011) or “Usable Knowledge” (Kersting et al., 2012) are notions which emphasise a phenomenological perspective, describing what teachers actually notice or what knowledge they use when analysing a classroom situation. These approaches in particular assume that a teachers’ professional knowledge might be larger than the knowledge s/he actually uses in the specific noticing or analysing process. But how are processes of noticing or analysing, i.e. processes which connect observations to specific elements of professional knowledge, initiated, and what guides and triggers these processes of noticing or analysing?

A model that aims at responding to this question has been suggested by Kuntze and Friesen (2018, see Figure 1.1). This model uses the perspective of mathematics teachers’ *analysing* (Kuntze, Dreher & Friesen, 2015; Friesen & Kuntze, 2014; cf. Seidel et al., 2011; Schneider et al., 2016). This notion encompasses core elements of the above-mentioned approaches (Kersting et al., 2012; Sherin et al., 2011; Berliner, 1991; Sherin et al., 2011; Dreher & Kuntze, 2015) and is understood as “an awareness-driven, knowledge-based process which connects the subject of analysis with relevant criterion knowledge and is marked by criteria-based explanation and argumentation” (Kuntze et al., 2015, p. 3214). In the sense of this definition, subjects of analysis can be classroom situations, or also tasks, lesson plans, or content units, for example.

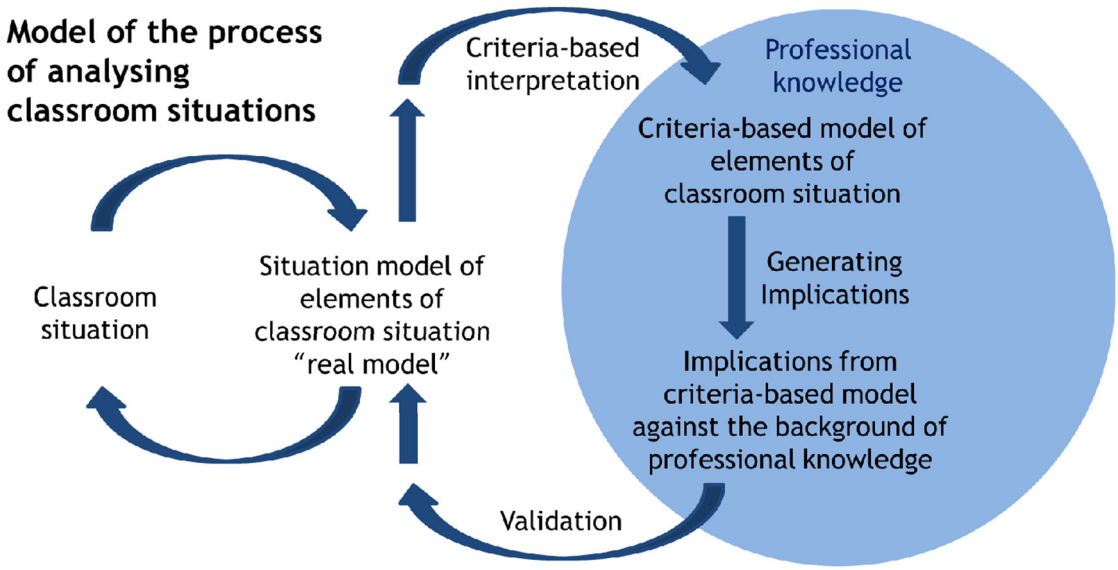


Figure 1.1: Model of the process of analysing (Kuntze & Friesen, 2016a, 2018)

The model presented in Figure 1.1 depicts the process of analysing classroom situations, it has a circular structure comparable to the modelling cycle (e.g., Blum & Leiss, 2005): The potentially circular process of generating (Fig. 1, left hand side) a situation model ("real model") of the classroom situation is a base for the teachers' interpretation of the situation based on criteria (Fig. 1, right hand side): Professional knowledge (including teachers' views, see model described in Kuntze, 2012) is the background, an analysing teacher draws on when describing observations in a criteria-based way. Against this professional knowledge background conclusions may be drawn by teachers analysing their observations. These conclusions can then be validated against the situation model of the classroom situation, which completes the interpretation cycle on the right hand side in Figure 1.1. Kuntze and Friesen (2018) assert that analogously to findings for the modelling cycle (Borromeo-Ferri, 2006), jumps between the phases shown in Figure 1.1 may occur. Moreover, parts of the process may take place unconsciously. Such unconscious processes may precede a more intense, explicit knowledge-based analysis process (e.g., with repeated cycles). Kuntze and Friesen (2018) consider the process as awareness-driven: In the model, simultaneous awareness for (possibly different) specific criteria continuously supports the possible criteria-based interpretation, the connection with professional knowledge and the validation of criteria-based implications.

Correspondingly, criterion awareness "keeps the cycle moving" and can be compared to a computer standby: If teachers dispose of criterion awareness

related to a specific criterion, this awareness helps to make observations in the classroom which fall into re domain of relevance of the criterion, and a knowledge-based interpretation process may be initiated in order to explore whether the observation can be explained according to the criterion, so that implications could be drawn. In case of mismatch during a validation step, either a deepened knowledge-based analysis related to the criterion could be carried out or a different criterion may take over and prevail in the further analysis (cf. Kuntze & Friesen, 2018). Criterion awareness has been defined as "a part of professional knowledge which influences the readiness and ability of teachers to use this professional knowledge element in instruction-related contexts" (Kuntze & Dreher, 2015, p. 298). Following this description, criterion awareness has the function of making professional knowledge accessible to teachers for situation analyses.
In general, Kuntze and Friesen (2018) assert that criterion awareness based on different criteria are in a competing relationship with each other for teachers' situation analysis. Different criteria (including individual criteria based on teachers' views) can be used for analysing classroom situations (e.g. Clausen, Reusser & Klieme, 2003), and it may happen that a criterion-based analysis cycle is initiated by at the same time neglecting an alternative criterion. Kuntze and Friesen refer to earlier findings supporting this competing relationship (e.g. Kuntze & Dreher, 2015).

Even if the model introduced above is particularly useful for describing key aspects of teacher expertise and teachers' professional learning, we would like to emphasise that the following considerations are not restricted to this particular theoretical approach.

1.3 Profession-related learning

For professional growth, learning processes of pre-service and in-service teachers play a key role. In this context, teachers' professional *learning* means the further development and construction of professional knowledge and/or teachers' growth in knowledge-based competences such as teachers' noticing, teachers' knowledge-based analysis and teachers' awareness. Alternatively, but also combinable with perspectives introduced above, teacher learning can be understood from a sociocultural perspective as a change in teachers' discourse over time (Wilson, Sztajn, Edgington, Webb, & Myers, 2015). In Mason's work (2011) noticing is conceptualised as "a movement or shift of attention" (p. 45) from "accounts of" to

“accounting for”. This movement implies an increase in sensitivity to the details of classroom situations, avoiding judgements, emotional content and generalities. While “accounts of” are free of theorising, emotional content, justification … “accounting for” refers to theorising and explaining. However, a moving in teachers’ discourse from “accounts of” to “accounting for” could be difficult without appropriate theoretical lenses to focus teacher’s attention on relevant details of the classroom situation. Levin, Hammer, and Coffey (2009) call for the use of tools and guidelines based on appropriate theoretical lenses to focus a teacher’s attention on relevant details of the classroom situation. Focusing on noticing students’ mathematical thinking, one of these theoretical lenses are the learning trajectories (Fernández & Choy, 2020). Learning trajectories provide prospective teachers and teachers with a cognitive model for thinking about a classroom situation and acting, and a specific mathematical language to describe students’ thinking (Edgington, Wilson, Sztajn, & Webb, 2016; Fernández, Sánchez-Matamoros, Valls, & Callejo, 2018; Ivars, Fernández, Llinares, & Choy, 2018; Sztajn, Confrey, Wilson, & Edgington, 2012) .

In this way, structured frameworks such as learning trajectories provide prospective teachers and teachers with useful guidelines to focus on important aspects of a classroom situation and to interpret them. In fact, Nickerson, Lamb, and LaRochelle (2017) highlight that “meaningfully analysing responses of [students and] interpreting and deciding how to respond to students’ mathematical ideas requires knowledge of students’ possible learning trajectories” (p. 393).

Ivars et al. (2018) and Ivars, Fernández and Llinares (2020) showed that the use of a learning trajectory (particularly, related to the part-whole meaning of fraction) as a framework to notice students’ fractional thinking (interpreting student’s mathematical understanding and deciding how to respond on the basis of students’ understanding) in a teacher training course helped prospective teachers to improve their noticing. Most of the prospective teachers who participated were more able to interpret the student’s understanding and provide activities focused on students’ conceptual progress at the end of the course. This was evidenced through prospective teachers’ discourse. Those who gave a more detailed discourse were more able to interpret students’ understanding and provide activities on the basis of student’s understanding. Along the course, evidence of a progression from a less detailed discourse (at the beginning prospective teachers did not give evidence of students’ understanding providing general comments or judgements) to entering a more detailed discourse focused on

students' mathematical thinking was found. Therefore, progress in prospective teachers' discourse (evidenced by the amount of details provided) is a sign of noticing students' mathematical thinking enhancement. This could be understood as an increase in sensitivity to the details of classroom situations, and therefore, a movement from "accounts of" to "accounting for", in order to express it in Mason's terms.

Under this perspective, enhancing noticing can be understood as a virtuous circle (Figure 1.2; Ivars et al., 2018; p. 13) in which learning trajectories can help prospective teachers in focusing their attention on specific details of the classroom situation, avoiding generalities or judgements.

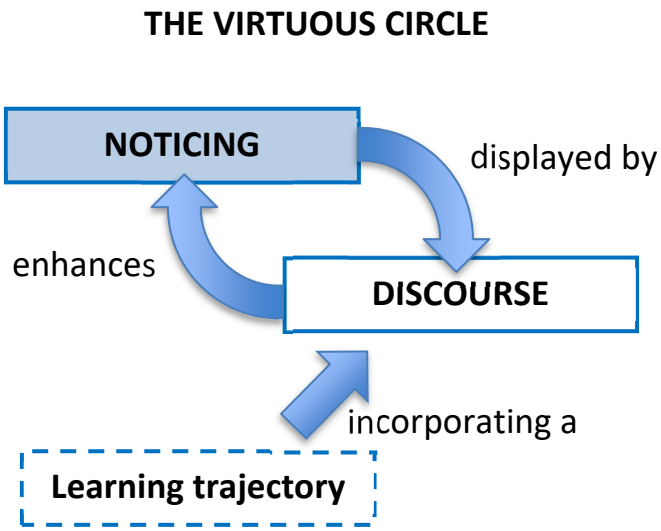


Figure 1.2. The virtuous circle of noticing (Ivars et al., 2018, p.13)

In a broader sense, different possible theoretical lenses or theory elements could replace the learning trajectories in Figure 1.2: Building up criterion knowledge related to dealing with representations, for instance (as well as related awareness) can be seen as a further example of a situation-overarching theoretical perspective which can enhance the teachers' noticing as well as developments in their discourse related to classroom situations (e.g. Friesen, Dreher & Kuntze, 2014; Kuntze & Friesen, 2018).

1.4 Vignettes as representations of practice – and theory

The profession of mathematics teachers brings a broad spectrum of requirements. This corresponds to a range of areas of mathematics teacher expertise, which can be described related to situation contexts - such as:

- Mathematics teachers have to be able to plan lessons, to analyse and select tasks for specific learning goals,
- they have to prepare and adapt learning environments;
- in class, they should be able to engage students in classroom discourse Mathematics teachers have to present explanations, examples, definitions;
- expert mathematics teachers need to keep track of student progress,
- explore and respond to what learners think,
- they should notice key events in classroom interaction, and
- be able to reflect on instructional practice,
- …

Even if the requirement contexts above are expressed as relatively general abilities or competences, the associated requirements mostly come in specific situations mathematics teachers should be able to master. Mathematics teacher education and professional development should prepare teachers for such requirements. Consequently, for the design of profession-related learning opportunities, the following questions are key: How can such situation contexts be accessed and made available in teacher education? How can they become learning opportunities for future teachers and support their professional growth?

Representations of practice (related to the teaching and learning of mathematics) are an answer to these questions. A valid reference to mathematics teachers' professional practice contexts is a means to exemplify such profession-related requirements, connecting learning opportunities with classroom situations, for instance, can create rich profession-related learning opportunities. A real classroom situation only happens once in a specific way - in contrast, representations of classroom practice can offer repeated access to instances of classroom practice. We understand the term *representation of (classroom) practice* in an analogous way to Goldin and Shteingold's (2001) notion of representations of mathematical objects: a representation of (classroom) practice is something that stands for this classroom situation, such as, for example, a drawing showing the situation, a cartoon, a narrative, a transcript, or a video showing the situation (Kuntze, 2018). Representations of classroom situations can be read or viewed several times. Like this, they afford a comparably open access to analysis and exchange among individuals, groups of teachers, or researchers.

We would like to point out that regardless of how a classroom situation is represented, the representation of practice should never be considered to be identical to the situation itself: This results from the fact that it appears as impossible to fully represent all individual perspectives of the persons involved in a classroom situation, for instance. Even videotaping classrooms does not lead to the situation itself: if a situation is videotaped, it shows the classroom through the camera perspective, and even though a video representation includes a large amount of context information (cf. Petko, Waldis, Pauli, & Reusser 2003), it is still barely impossible to capture all potentially meaningful context aspects. Video is “but a tool” (Seago, 2004, p. 263) - also videotaped classrooms can only stand for the classroom situations they represent.

Vignettes and their double representational function

Under this scope, *vignettes* are understood as representations of profession-relevant practice (Buchbinder & Kuntze, 2018). Through showing aspects of a classroom situation, for instance, vignettes stand for specific aspects of practice contexts and thus represent those.
Vignettes can take different forms and reflect different ways of representing: Classroom situations, for instance, can be represented in video, text or cartoon formats, among others, or combine two or several of these formats. In most cases, vignettes are accompanied with a question, in which teachers can be asked to comment in a criterion-based way, to report their observations or suggestions, to reflect on the vignette situation, to prepare a discussion, or to engage in other activities. In particular, vignettes offer possibilities of eliciting teachers’ analysis of classroom situations in systematically designed ways (Skilling & Stylianides, 2020; Buchbinder & Kuntze, 2018; Fernández et al., 2018; Ivars, Fernández, & Llinares, 2020).

Vignettes are not only representations of practice, *they also represent theory* (Kuntze & Friesen, 2020): This results from the production process of vignettes, in which particular elements of theory, for example intended observation criteria or goals of a professional development activity mostly play a role. For instance, when a classroom interaction is designed so as to show an example of a more general known phenomenon, or even when a video sequence is selected from more extensive authentic video material according to a set of criteria or a specific aim (cf. e.g., Kuntze, 2018; Nardi, Healy, Biza & Fernandes, 2018) - then elements of theory interfere with the situation context(s), so that the vignette carries these theoretical elements and hence stands for them as well.

The consequence is that vignettes have a *double representational function* both for practice contexts and for theory. In this way, vignettes mediate between mathematics education theory and practice - this results in their high potential for professional learning.

Vignettes in different forms

Vignettes as representations of both practice and theory (related to the teaching and learning of mathematics) can represent various aspects of quality teaching or breaches of classroom-related norms. They can be purposefully selected, transformed from authentic material or designed (Skilling & Stylianides, 2020; Kuntze & Friesen, 2020, 2017; Fernández et al., 2018; Mesa & Herbst, 2011; Seidel et al., 2011). Vignettes can provide rich learning opportunities as they can foster teachers' ability to analyse and reflect on classroom situations without the pressure to act - alternatively, time pressure can be implemented in vignette-based work as a simulation of real-classroom time pressure. The choice of a vignette format should therefore carefully consider goals of professional learning, but also the target group. Perceived authenticity of the vignettes, motivation, immersion and resonance in vignette-based work play important roles for teachers' engagement with representations of practice. These considerations about different ways of constructing and using vignettes will be foregrounded in the following.

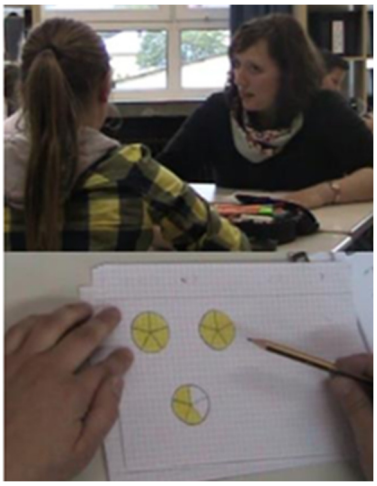
2. Different ways to construct and use vignettes

When developing vignettes, some important decisions need to be considered: In particular (1) capturing the vignette context or meaning and (2) portraying the vignette 'story' in a way that resonates with those responding to it play key roles. Furthermore, it needs to be considered whether to use existing information or to construct new material that captures relevant content which is meaningful to the participants. Some vignettes are based on real-life events that are then turned into a story (Bradbury-Jones et al., 2014; Jeffries & Maeder, 2005; Poulou, 2001). Some vignettes are based on existing literature and conceptual frameworks (Simon & Tierney, 2011), or on personal or professional experiences (Veal, 2002), or combinations of these.

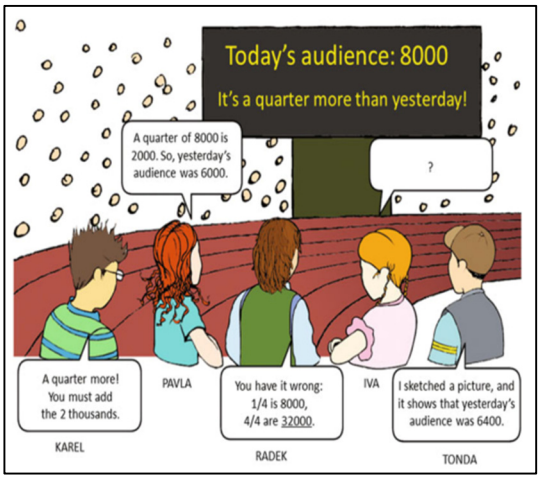
How the vignette's 'story' or 'narrative' is portrayed to potential participants is vital so that it resonates with the user or responder. On the one hand, vignette information should be concrete enough to approximate the reality of a situation but, on the other hand, be abstract enough to allow participants to form their own interpretations, understandings and beliefs "on topics very familiar to them" (Poulou, 2001, p. 58). Therefore, vignettes are different to 'made up' stories because they are based on events, episodes, characters, and descriptions of real-life (and classroom) situations (Skilling & Stylianides, 2020).

An important characteristic of vignettes is that they can represent one aspect or several aspects of a classroom situation, but not all the characteristics of a class (Buchbinder & Kuntze, 2018). This reduction of information makes the vignettes useful instruments in the professional development of teachers since they allow teachers to focus the attention on those aspects of the practice that are intended to be the object of learning.

Vignettes can be designed in different formats: in video recordings of real classroom situations (van Es & Sherin, 2008), in written form such as student responses to different problems or dialogues between the teacher and the students solving different problems (Fernández et al., 2018; Ivars, Fernández, & Llinares, 2020) or in animations or cartoons (Herbst & Kosko, 2013; Samoková, 2018). Figure 2.1 presents some examples from vignettes in different formats.



(Friesen, 2017, p. 60)



(Samkova, 2019)

André and the pyramid

Imagine you are teaching a class about three-dimensional geometric figures. As the students work on exploring how they would describe what a square-based pyramid is to someone who doesn't know, you move around the class to observe their strategies. You notice many are counting faces, edges and vertices. André, who is blind, has been working with materials, such as 3D solids. He offers this description. [*Video clip follows*]

Questions:

What is André proposing as a description of a square-based pyramid?

What do you do next?

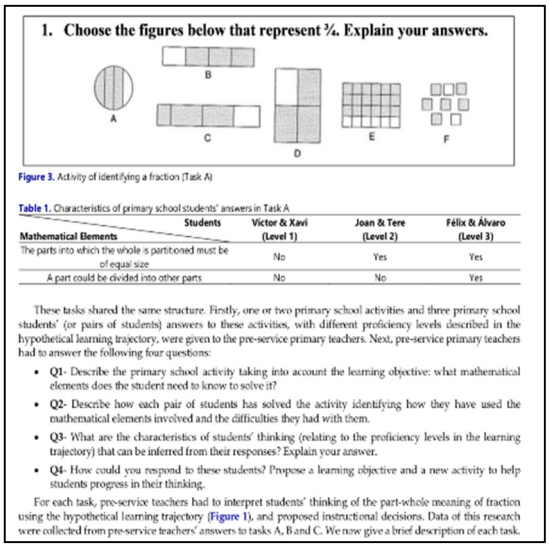
What do you think are the issues in this situation?

What prior experience do you have in dealing with these issues?

What prior experience do you have in supporting the mathematical learning of blind students in your classroom?

How confident do you feel about including blind students in your classroom?

(Nardi, Healy, Biza, & Fernandes, 2016; 2018)



1. **Choose the figures below that represent ¾. Explain your answers.**

A B C D E F

Figure 3. Activity of identifying a fraction (Task A)

Table 1. Characteristics of primary school students' answers in Task A

Mathematical Elements \ Students	Victor & Xavi (Level 1)	Joan & Tere (Level 2)	Félix & Álvaro (Level 3)
The parts into which the whole is partitioned must be of equal size	No	Yes	Yes
A part could be divided into other parts	No	No	Yes

These tasks shared the same structure. Firstly, one or two primary school activities and three primary school students' (or pairs of students) answers to these activities, with different proficiency levels described in the hypothetical learning trajectory, were given to the pre-service primary teachers. Next, pre-service primary teachers had to answer the following four questions:

- Q1- Describe the primary school activity taking into account the learning objective: what mathematical elements does the student need to know to solve it?
- Q2- Describe how each pair of students has solved the activity identifying how they have used the mathematical elements involved and the difficulties they had with them.
- Q3- What are the characteristics of students' thinking (relating to the proficiency levels in the learning trajectory) that can be inferred from their responses? Explain your answer.
- Q4- How could you respond to these students? Propose a learning objective and a new activity to help students progress in their thinking.

For each task, pre-service teachers had to interpret students' thinking of the part-whole meaning of fraction using the hypothetical learning trajectory (Figure 1), and proposed instructional decisions. Data of this research were collected from pre-service teachers' answers to tasks A, B and C. We now give a brief description of each task.

(Ivars, Fernández, Llinares, Choy, 2018)

Figure 2.1: Different formats for designing vignettes: video, written or animations/ cartoons

From research, it has for example been shown that the participation in a professional development in which the participants worked with video vignettes enhanced in-service teachers' analyses related to discourse and student-centred argumentation (Kuntze, 2018). Regarding the animations/cartoons format, the use of Concept Cartoons focusing on students' (mis)conceptions and their possible sources helped pre-service primary school teachers acquaint with varied mathematical ideas of hypothetical primary students, and thus the Concept Cartoons accompanied by a list of indicative questions helped the pre-service teachers reason about selected mathematical topics (Samková, 2018a), or challenge their pedagogical content knowledge in mathematics (Samková, 2018b, 2019).

Considering text-based vignettes, it has been shown that these vignettes as tools that focus pre-service teachers' attention on important aspects of students' understanding, can help them develop a professional vision (a review in Fernández et al., 2018; Ivars, Fernández, & Llinares, 2020; Llinares, Fernández, & Sánchez-Mamatoros, 2016; Sánchez-Matamoros, Fernández, & Llinares, 2019). This research has further shown that some characteristics that help teachers focus their attention on the important aspects of student's understanding are: the use of guiding questions and the use of theoretical lenses (Fernández & Choy, 2020) such as the use of hypothetical learning trajectories (Ivars, Fernández, Llinares, & Choy, 2018).

Several studies found that different formats (e.g. text, video, cartoon) are comparably useful for engaging pre-service and in-service teachers with representations of classroom practice (Herbst & Kosko, 2013; Friesen & Kuntze, 2018). Of course, vignette methods may involve using a combination of these formats. However, for each of these formats there are certain features that should be considered by the vignette developer in order to facilitate its use in the intended context. These pertain to particular characteristics and elements (a number of which are discussed in the paper by Skilling and Stylianides (2020). In brief, considering the length, language and delivery of the vignette in respect of the participants and expected responses are important. Below are some suggestions for different vignette formats.

Written formats

- Length should be within 50-200 words to maintain interest and reduce fatigue
- Language used should be appropriate for the intended audience
- Being read by or to participants requires decisions about assisting the participant to access the narrative but may also induce bias
- If questions of the vignette require written responses consider the demands on participants
- The selection of a wide variety of students' responses to an activity/or different activities that shows different levels of the mathematical concept understanding can be involved (Fernández et al., 2018).
- The use of guiding questions and theoretical lenses to help pre-service teachers focus their attention on specific aims (Fernández et al., 2018; Ivars et al., 2020) can be useful
- Text formats can take very different forms (e.g. transcripts, narratives, …), so that related decisions need to be made

Cartoon/images – visual vignettes

- Combination of texts and images should consider text to image ratio, and aspects such as layout which may be complex
- Number of pictures should be considered, e.g. use one individual snapshot or more like a comic strip
- Different sets of indicative questions may have different aims (motivation, assessment, knowledge development) (see Samkova, 2019, 2020)

Video

- Generally video snippets are recommended lasting 1-4 minutes. Friesen and Kuntze (2016a, 2018) used video vignettes lasting one and a half minutes that could be paused and re-viewed as required
- Friesen and Kuntze (2016a, 2018) also report that videos tend to highlight distracting elements such as background material and clothing of actors which may detract the participants attention from the focus of the vignette narrative itself. This effect can play a potentially distracting role, but it can also be intended in order to approach the complex demands of authentic classroom situations.

3. Authenticity, resonance & Co. - Reflecting on vignettes and on their use for stimulating professional learning

In this activity, you are invited to reflect on a set of example vignettes from different contexts of pre-service teacher education and in-service teacher professional development. This activity can be done online through the link included in Figure 3.3. Through this Link which directs you to one of the vignettes (see Figure 3.2), you can also find the complete set of the other vignettes and questions related to their authenticity, resonance, etc. Such questions are also displayed in Figure 3.1.

Moreover, you are invited to reflect on culture-specific aspects of the vignettes and the situation contexts represented in them. For each of the vignettes, you can use the following key questions to stimulate reflection and to engage in discussion with your colleagues:

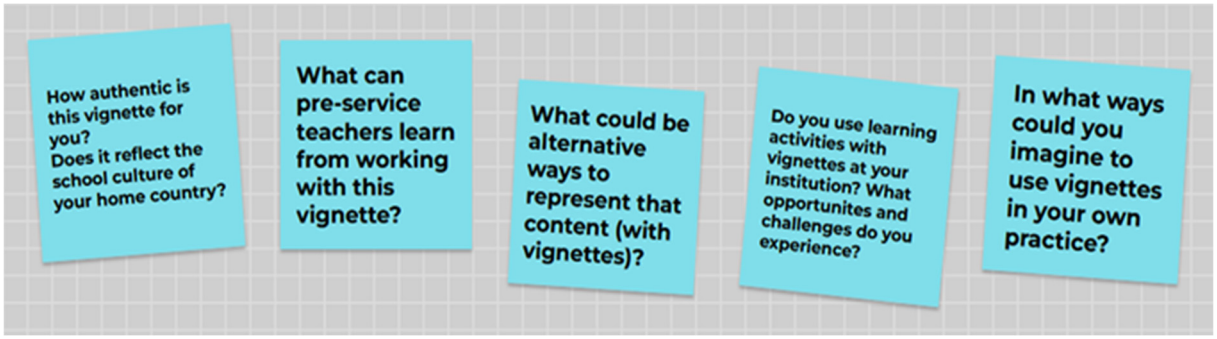


Figure 3.1: Reflection questions related to authenticity, resonance & Co., as well as to culture-specific backgrounds

The vignettes will be presented and commented in more detail in the following section (section 4).

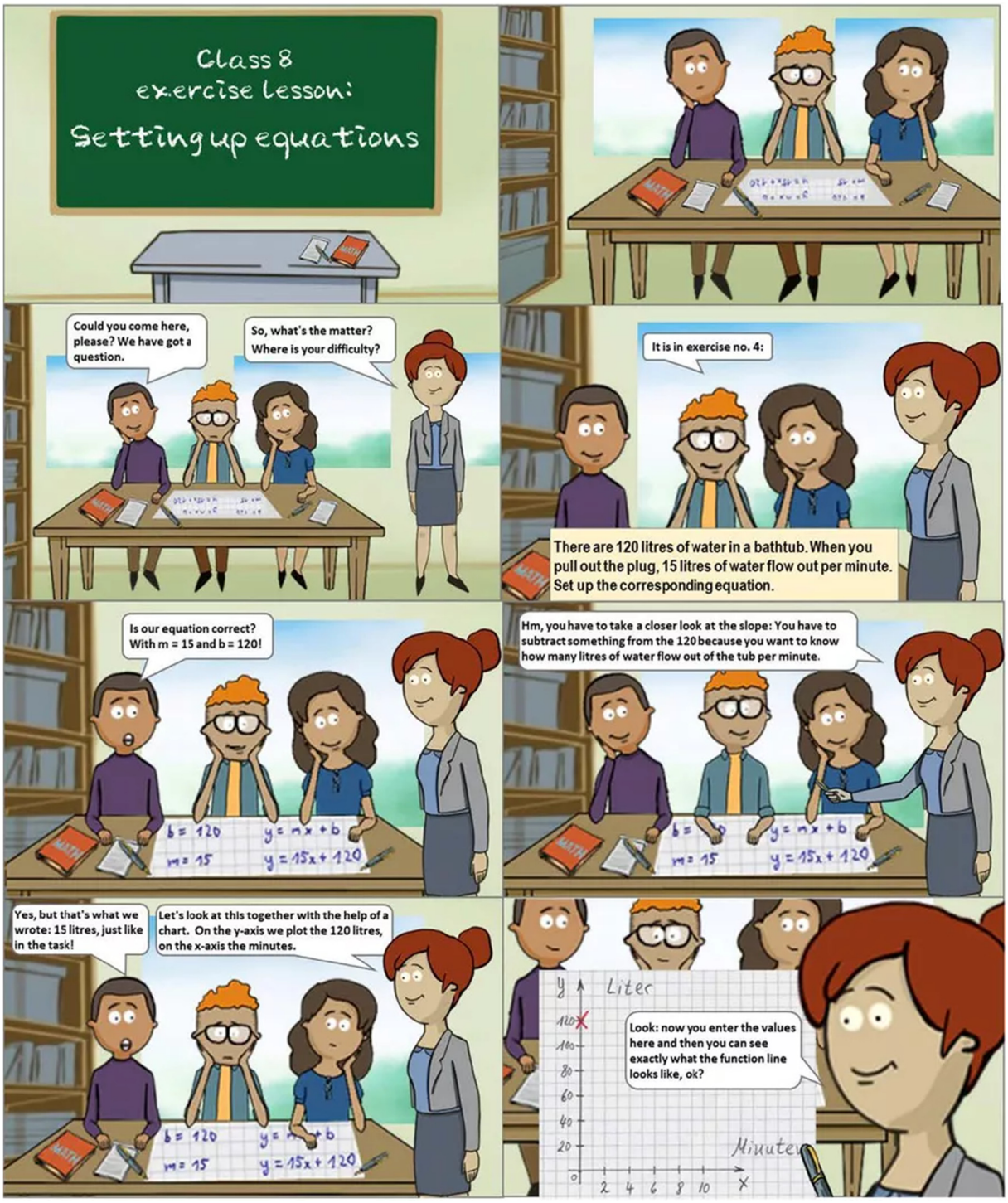


Figure 3.2: This sample vignette deals with the use of multiple representations of linear functions when setting up equations from a given situation context. If you would like to engage with this vignette sample in more depth, you are invited to follow the link in Figure 3.3 or scan the code for accessing the activity.

Activity

How do you see this classroom situation?

We would like to invite you to analyse this vignette and to share your analysis with us in an anonymised online survey.

Follow the link and take part in the activity!

http://www.coreflect.eu/activity.html

Figure 3.3: Link to the activity related to the vignette in Figure 3.2

4. Topics, target groups, profession-related learning objectives: vignette examples

The key goal of this section is to take an exemplary look at a set of different vignettes, in order to open up the consideration of the following guiding questions:

- Which vignette design is most suitable to represent aspects of teaching expertise addressed in a particular teacher education course?
- How can preservice teachers be facilitated in analysing and reflecting on a classroom situation, e.g. by providing prompts or a specific framework?

In order to support the awareness of the potential role of individual vignettes in a framework of profession-related learning opportunities, their topics and target groups, as well as their learning objectives, we will in the following showcase a set of vignettes from the project coReflect@maths. These six vignettes were designed and used for mathematics teacher education

- in different cultural contexts and
- addressing different learning objectives and aspects of teaching expertise.

Each vignette will be preceded by an overview on target group(s), profession-related learning objectives and mathematics-related topic. Further explanations related to the specific vignette examples will be given as well, together with each vignette.

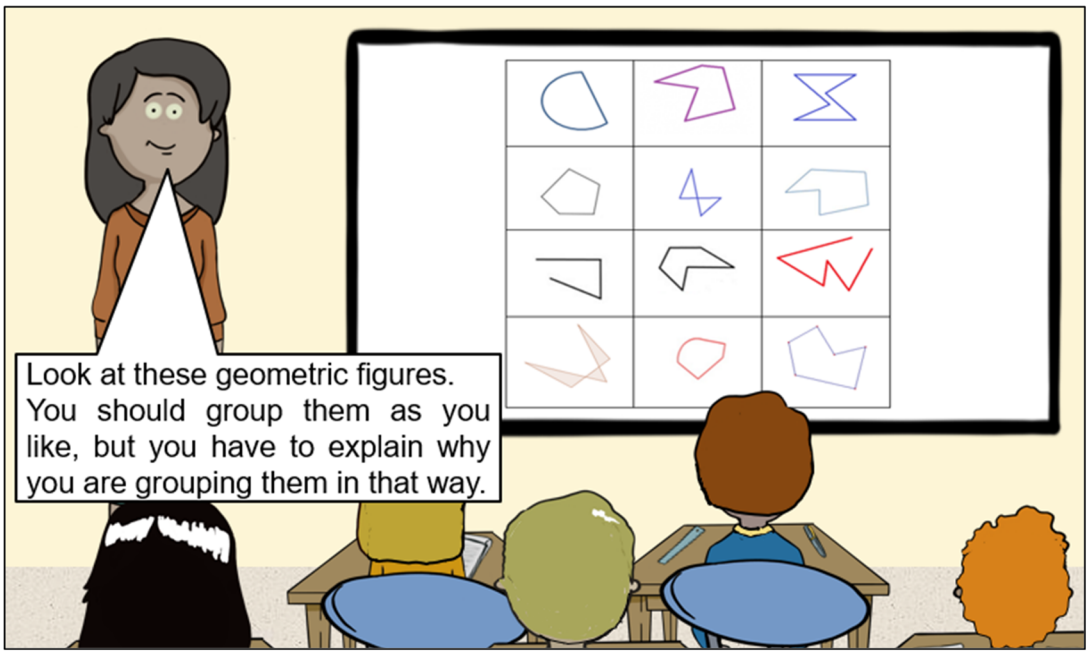
4.1 Analysing students' geometrical thinking: A vignette on polygons and their properties

Target group:	Pre-service primary teachers
Profession-related learning objectives:	Core Practice, noticing, Interpreting students' mathematical thinking
Topic:	Geometrical thinking

Ana is a primary school teacher in a 2nd grade classroom. The goal of the lesson is "recognising and reasoning with the attributes of geometrical figures to construct the polygon concept"

She proposes the following activity to her pupils to be solved in small groups.

Next, she checks how her students are solving the activity.



Student 1

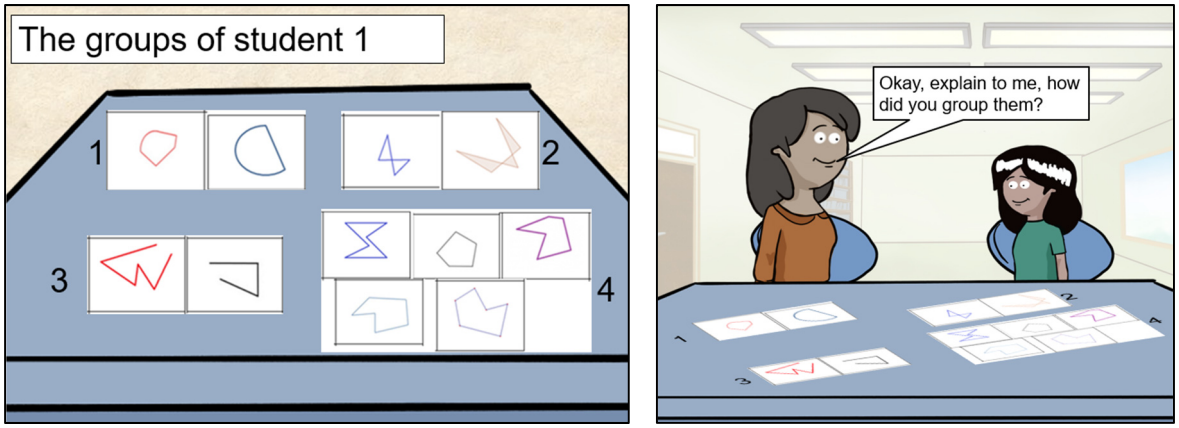
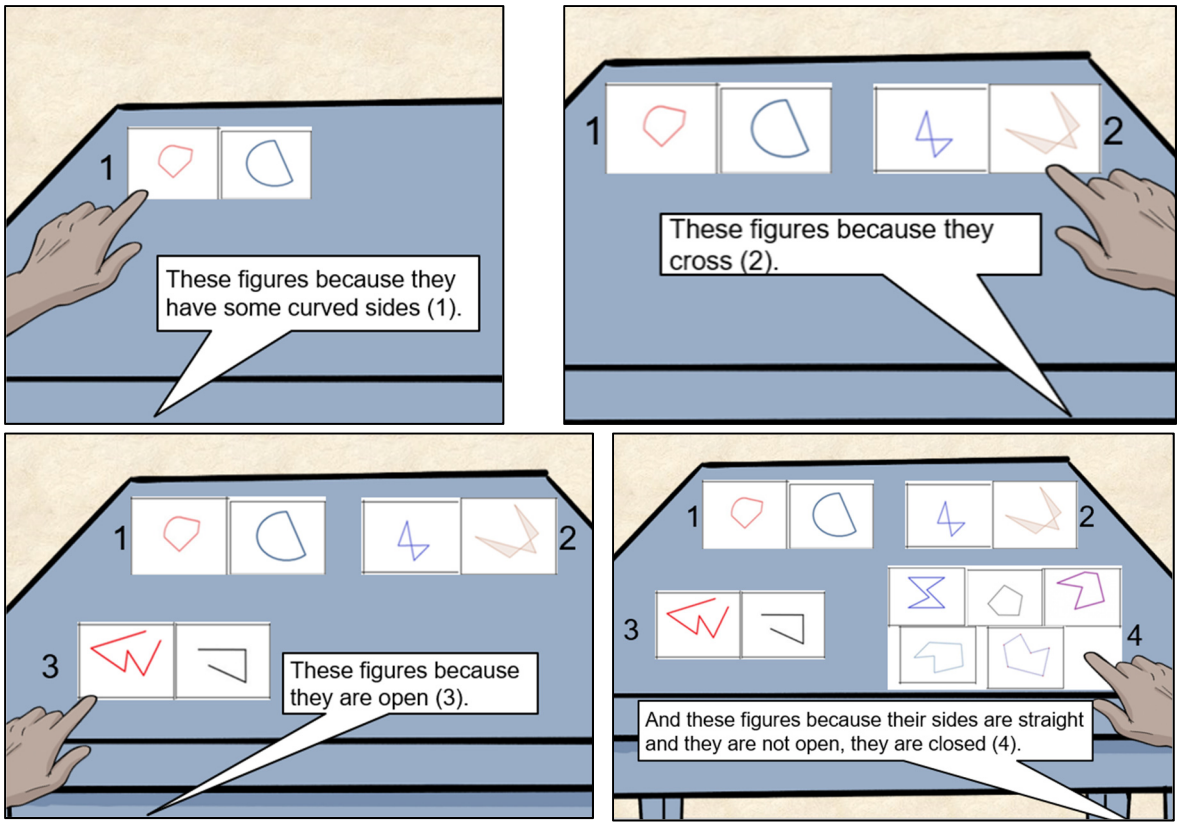


Figure 4.1 (Part I): Vignette on reasoning with attributes of polygons



Student 2

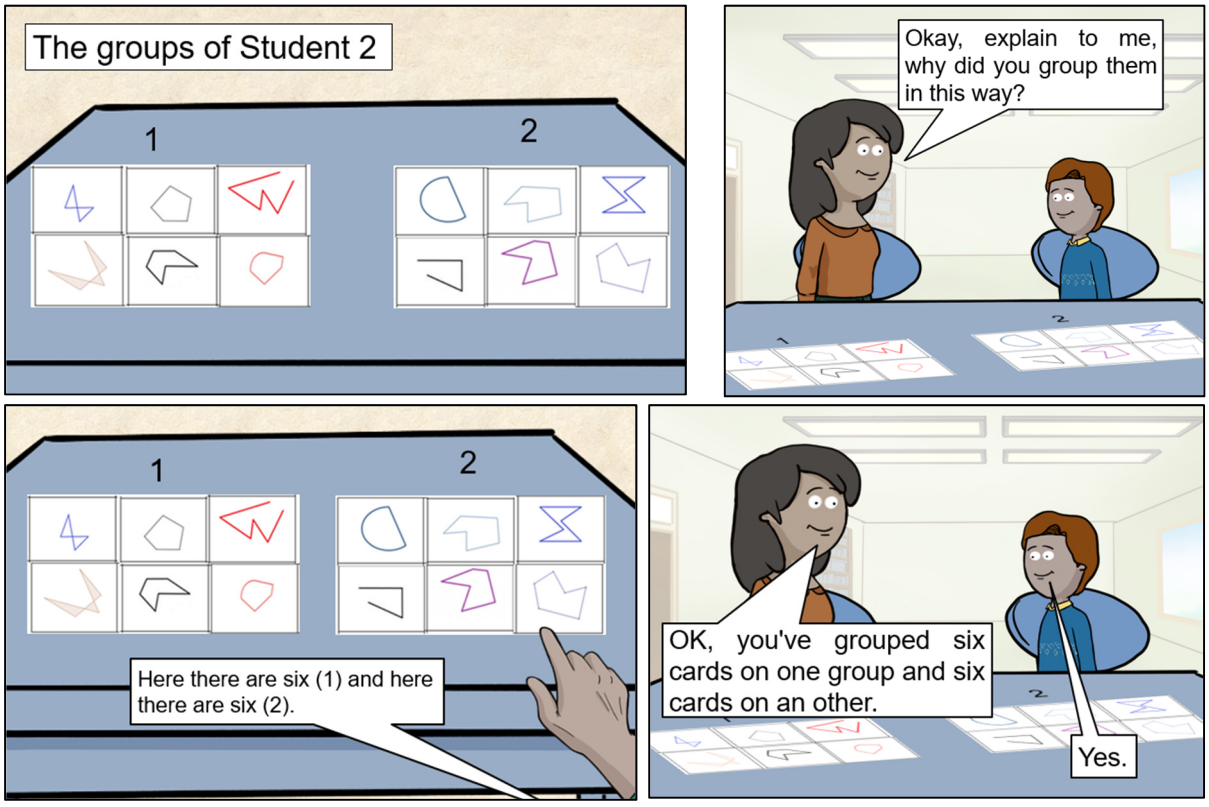
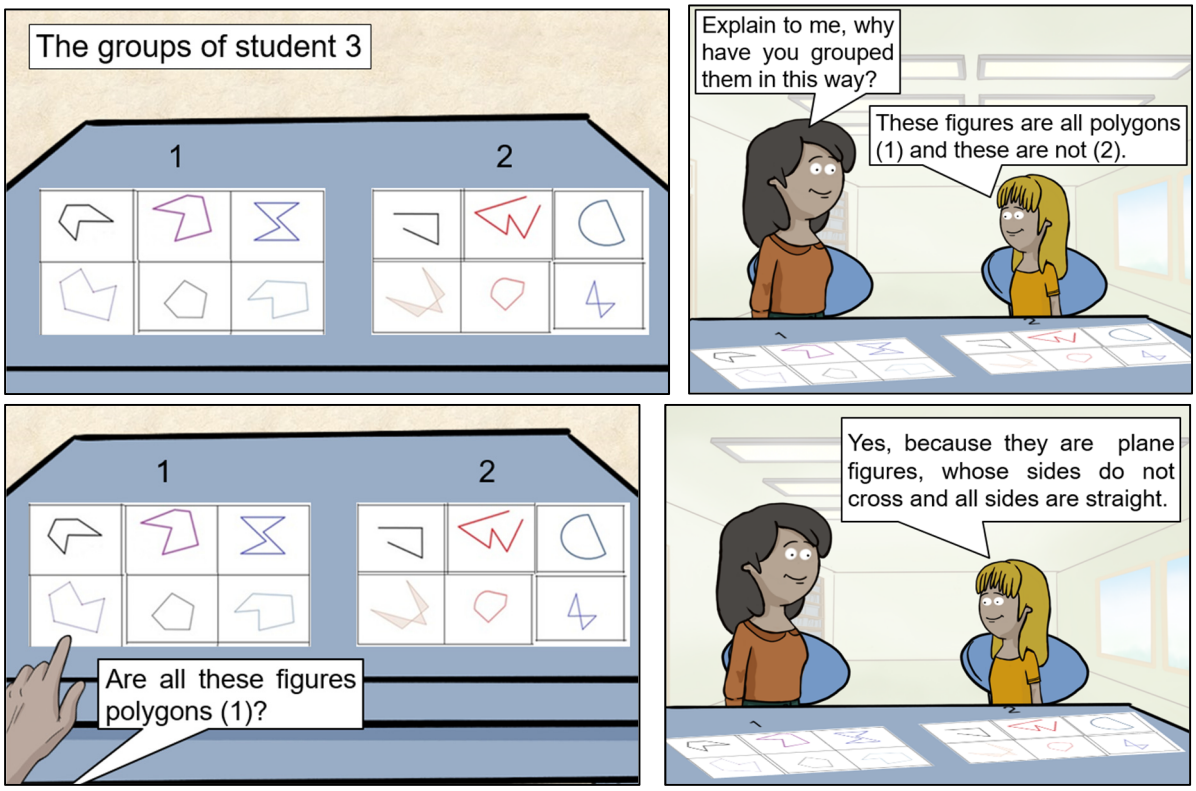


Figure 4.1 (Part II): Vignette on reasoning with attributes of polygons

Student 3



QUESTIONS

- **Q1**- Describe the activity. What are the geometrical elements and processes involved in the activity?
  - Identify the characteristics of the examples used by the teacher
- **Q2**- Describe how each student has solved the activity, identifying:
  - How has he/she used the *geometrical elements and processes*? which difficulties has he/she had?
  - At which level of geometrical thinking development would you place him/her? Explain your answer.
- **Q3**- Considering the level at which you have placed each child, define a learning objective for the next lesson and propose an activity (or modify the one initially proposed by Ana) to help the students progress in the development of geometrical thinking.

Figure 4.1 (Part III): Vignette on reasoning with attributes of polygons

This vignette belongs to a course concept (for vignette-based course concepts see also below in section 9) on Geometrical Thinking (topic) for pre-service primary school teachers (grades 1-6) (target group) to enhance pre-service primary teachers' noticing of geometrical teaching situations (profession-related learning objective). The specific learning goals are: attending to, interpreting and making decisions about students' geometrical thinking; analysing instructional task material and textbook pages (and interactive resources); analysing students-teacher interactions (language in

classroom); and designing instructional tasks (lesson plan) to promote the development of students' geometrical thinking.

This vignette describes a teaching situation based on students' geometrical thinking of recognising and reasoning with attributes of figures. Pre-service primary school teachers are expected to attend to primary students' reasoning process, to interpret and make decisions about this situation. The vignette shows three primary students' answers to a geometrical task. The primary teacher asks the students to group cards with examples and non-examples of polygons. The students' answers show different features of the primary students' understanding of the polygon concept.

To analyse this teaching situation, the pre-service primary teachers have a document with information on the development of students' geometrical thinking and about the features of geometrical tasks (Battista, 2012). There are three guiding questions about attending to, interpreting, and deciding according to the situation. The guiding questions are:

Q1- Describe the activity. What are the geometrical elements and processes involved in the activity? Identify the characteristics of the examples used by the teacher.

Q2- Describe how each student has solved the activity, identifying:

- How has s/he used the geometrical elements and processes? which difficulties has s/he had?
- At which level of geometrical thinking development would you place her/him? Explain your answer.

Q3- Considering the level at which you have placed each child, define a learning objective for the next lesson and propose an activity (or modify the one initially proposed by Ana) to help the students to progress in the development of geometrical thinking.

The guiding question Q1 is based on attending to the situation, where the elements and processes involved allow students to identify and use attributes of geometrical figures (as for example, open/closed shapes; crossed or uncrossed sides; straight or curved sides; classes of polygons such as concave/convex or according to the number of sides, etc.). The teacher's demand for justification can help determine how students generate detailed explanations of classifications. So, this task allows students to use attributes explicitly to make classifications and begin to detail explanations of classifications and recognise at what level of development of understanding of geometric figures each student is at.

Question Q2 is related to interpreting the situation, where pre-service primary school students have to characterise the answers interpreting the

students' understanding of geometric figures. For example, in the example of Student 3, the student groups the shapes explicitly using the definitions, she does not provide unnecessary attributes when defining polygons and groups polygons and non-polygons clearly and provides the definition of polygon without including irrelevant or perceptual attributes illustrating the Level 3 of understanding of geometric figures.
Question Q3 is based on making decisions about the situation. Pre-service primary school students have to design a task to help students progress in the development of geometric thinking. For example, to the "Student 3", a task to perform classifications using two attributes. The materials for this task would be geometrical figures cut out on cardboard (or geometrical figures drawn in playing card format), and a cardboard on which a grid is drawn indicating the classification criteria.

4.2 Mathematical content and pre-service teachers' beliefs: A vignette about the congruence of triangles

Target group:	Secondary Mathematics trainee teachers (for teaching ages 12-15years)
Profession-related learning objectives:	- Subject and pedagogical knowledge about congruence for the case of triangles - Eliciting beliefs about teaching approaches for promoting student understanding of congruency
Topic:	Congruent triangles

The 'Congruent Triangle' vignette (Figure 4.2) portrays a classroom situation with two scenes. The first scene depicts the teacher presenting a non-routine task to the class. The second scene shows two students discussing the task, their prior knowledge and asking if it has to do with defining triangles. The teacher responds and asks what information is needed to prove two triangles are congruent. The four questions that accompanied the vignettes are listed below in Table 1 with their focus for eliciting the pre-service teachers' content and pedagogic knowledge, beliefs about deep learning, approaches to teaching, strategy use and reflection on learning. The questions also required responses from different positions or characters. For example, the participants' view was asked in question 1; the

students' view was sought in question 2 and 3; and the teachers' view was sought in question 4.

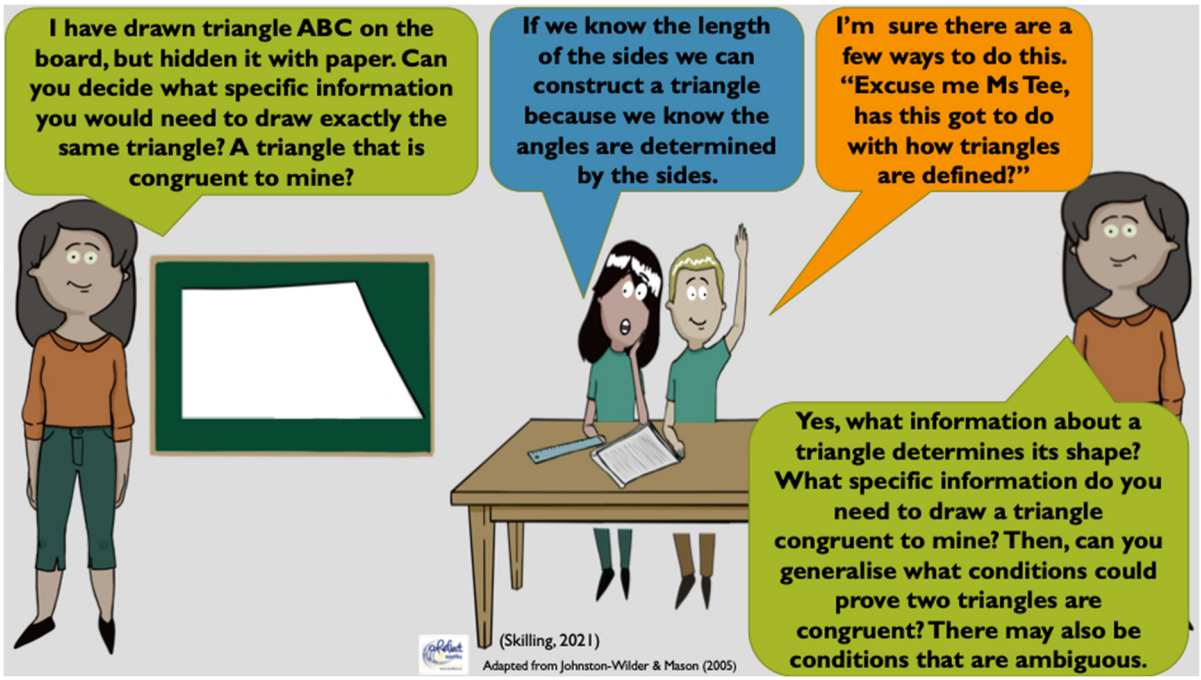


Figure 4.2: Congruent triangle vignette

Questions for the 'Congruent Triangle' vignette	Foci – used for coding
1. What specific information might the students need to draw Ms Tee's triangle?	Subject knowledge
2. What conditions do you think the students might establish first… then second and so on? Please explain why you think this.	Subject knowledge Pedagogical knowledge
3. What ambiguous condition might the students identify and how would you explain this if it was raised in a lesson you were teaching?	Subject knowledge Pedagogical knowledge
4. Rather than stating the conditions for establishing congruent triangles, Ms Tee asked the students to identify what information they needed about triangles and to prove congruence. By taking this approach what type of thinking processes do you think Ms Tee was aiming to promote which are important in the context of geometry	Deep learning

Table 4.1: Questions and coding for the 'Congruent Triangle' vignette

The vignette was given to a group of pre-service teachers in the first part of a year-long course. The vignette was introduced at the beginning of a university teaching session about using tasks including non-routine tasks so that their responses could be collected prior to the whole class discussion and the seminar presentation. The vignette was sent electronically and the pre-service teachers were asked to respond to open-ended questions accompanying the vignette which aimed to elicit their content and pedagogical knowledge in respect of congruence for triangles and their beliefs about ways to approach teaching this topic. Textbooks often portray this topic as one where certain conditions are 'learned' (about the sides and angles). The purpose of this vignette was to provoke deeper thinking about understanding congruence and also provoke thinking about different ways of approaching the teaching of this concept. The aims are reflected in the two research questions:

1. What forms of professional knowledge do pre-service teachers discern?
2. In what ways, do fictitious vignettes provide opportunities for pre-service teachers to reveal their professional knowledge and imagined practice?

4.3 Analysing how multiple representations are dealt with in the classroom: A vignette on setting up equations

Target group:	Mathematics teachers at different levels of experience
Profession-related learning objectives:	- Learn to analyse students' mistakes - Learn to analyse teachers' use of representations
Topic:	Using multiple representations of linear functions

The situation represented in the cartoon in Figure 4.3 shows a typical classroom activity with students working on their tasks and with a teacher walking from desk to desk answering questions and providing support. In such a setting, it is particularly important for the teacher to notice the students' understanding and typical mistakes (as shown in the representation they use) in order to be able to adapt their support or explanation to the students' thinking. Since changing between different representations is important for developing understanding on the one hand

but also quite challenging for the students on the other, the teachers' support in connecting and linking different representations of the same linear function is very important (Friesen, 2017; Friesen & Kuntze, 2016; Duval, 2017).

Accordingly, key objectives addressed by this vignette are: (1) learn to analyse students' mistakes and (2) learn to analyse teachers' use of multiple representations when supporting students in their use of different representations of functions (here: graph, equation, situation context). It can be interesting to discuss this vignette with colleagues at different levels of experience, e.g., student teachers and more experienced practitioners.

Figure 4.3: Vignette on setting up equations of linear functions (see Figure 3.2 for a cartoon in larger size)

4.4 Evaluating learners' thinking: A vignette on reasoning with fractions

Target group:	Prospective primary school teachers
Profession-related learning objectives:	- Investigating content knowledge about fractions (the first and second questions below the picture) - Investigating pedagogical content knowledge about fractions (all the questions) - Developing pedagogical content knowledge: learning to analyse students' solutions and assess them in a formative way, getting acquainted with a widespread misconception (Alice)
Topic:	Relative change given by a fraction

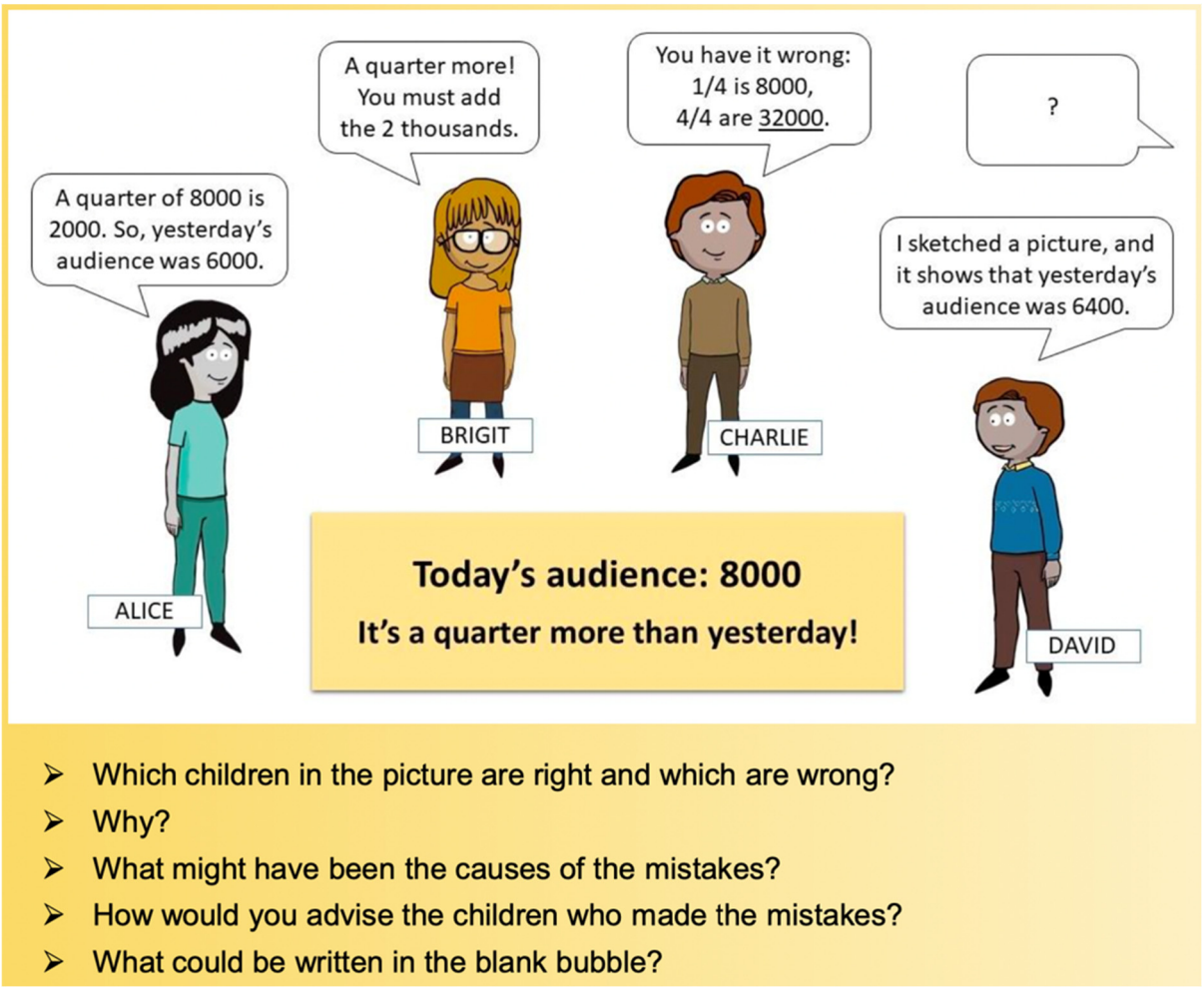


Figure 4.4: Vignette on reasoning with fractions

This vignette (Fig. 4.4) is a one-picture cartoon. In the yellow frame, it displays a mathematical task about a change expressed by a fraction. Various opinions on solving this task are written in the bubbles. The task was a low-success task in a state maturity exam several years ago (for more details, see Samková, 2018a); only 1/3 of the examined students solved the task correctly. The most frequent incorrect solution is represented in the bubble of Alice. Various assignments of the vignette allow to achieve various research goals – focusing on content knowledge, pedagogical content knowledge, both at the same time (for more details, see Samková, 2019).

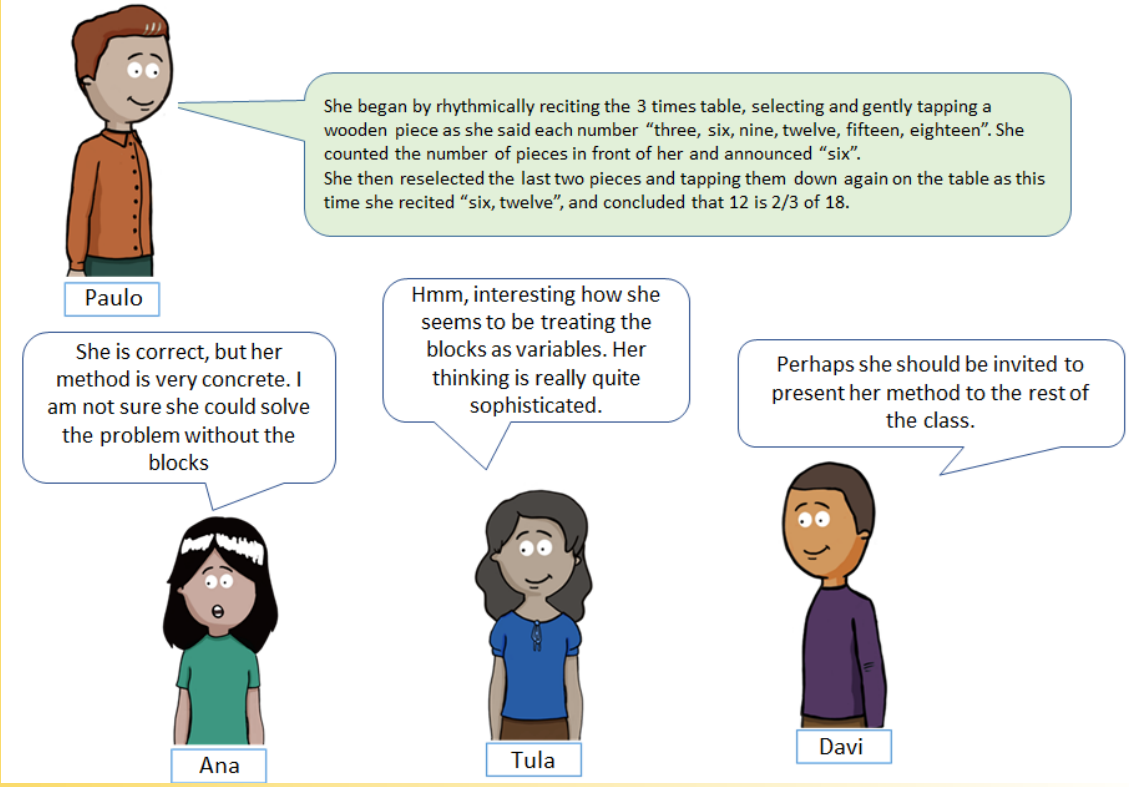
4.5 Opportunities for extending the horizon in profession-related learning with focus on students with special needs: A vignette focusing on a student's approach to multiplicative structures

Target group:	Pre-service primary teachers
Profession-related learning objectives:	Core Practice, noticing, interpreting students' mathematical thinking
Topic:	Multiplicative structures

This vignette combines a text description with a cartoon in which Vitória, a 10-year old blind student and the key "actor" in the vignette, is not visible. Beyond the highly interesting content core related to Vitória's mathematical thinking and acting, the vignette design adds further opportunities for reflecting on "seeing" what happens in mathematical learning, "observing" individual approaches, and interpreting observations in order to evaluate individual approaches. This vignette is an example how a focus on students with special needs open up ways of questioning narrow conceptualisations related to the learning (and teaching) of mathematics, of digging deeper into the foundations of mathematical concepts and also of becoming aware of the limitations of how a learning situation can be perceived by a teacher.

Vignette: Vitória's Method

Vitória, a 10-year old blind student at a mainstream school in the Brazilian state of Sao Paulo, was not confident in reading or writing Braille. At the beginning of the school year, her support worker, Paulo, had made her some small wooden pieces that fitted comfortably in her hand, which she often used when solving mathematics problems. Paulo observed her solving the calculations 2/3 of 18, 3/5 of 30 and 3/4 of 24, using the same method. He described to three other teachers her method for 2/3 of 18 and asked for their comments.



- How would you evaluate Vitória's method and what would you do next to support her developing ideas about multiplicative structures?
- How would you evaluate the comments of Ana, Tula and Davi and what do you think they might prioritise as their next steps to support her learning?
- How do you think that the mathematics she is learning is affected by the way Vitória sees with her hands and ears?

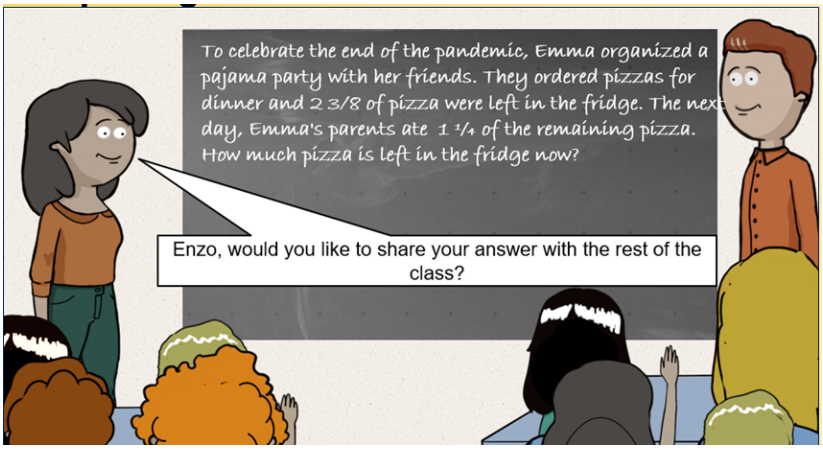
Figure 4.5: Vignette on a student's approach to multiplicative structures

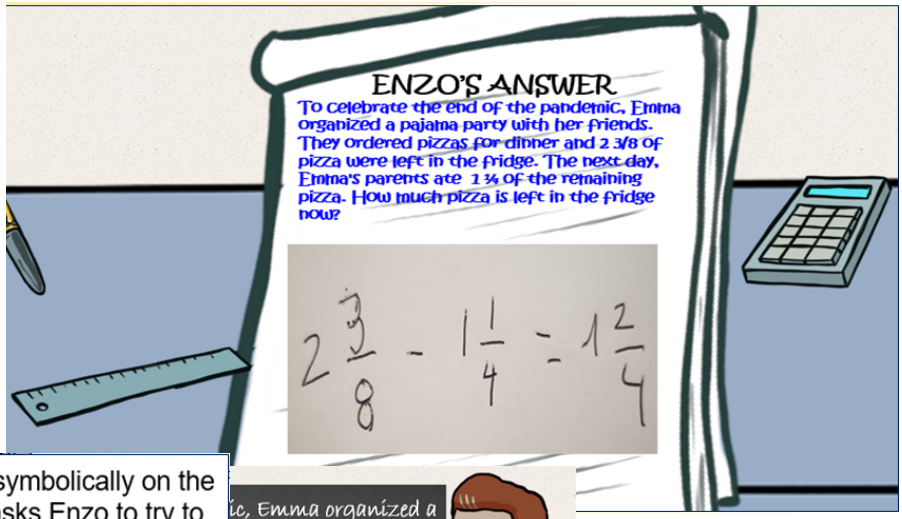
4.6 Providing help and dealing with mistakes: A vignette on subtracting mixed numbers

Target group:	Preservice primary teachers
Profession-related learning objectives:	Core Practice, noticing, Interpreting students' mathematical thinking
Topic:	Fraction operations

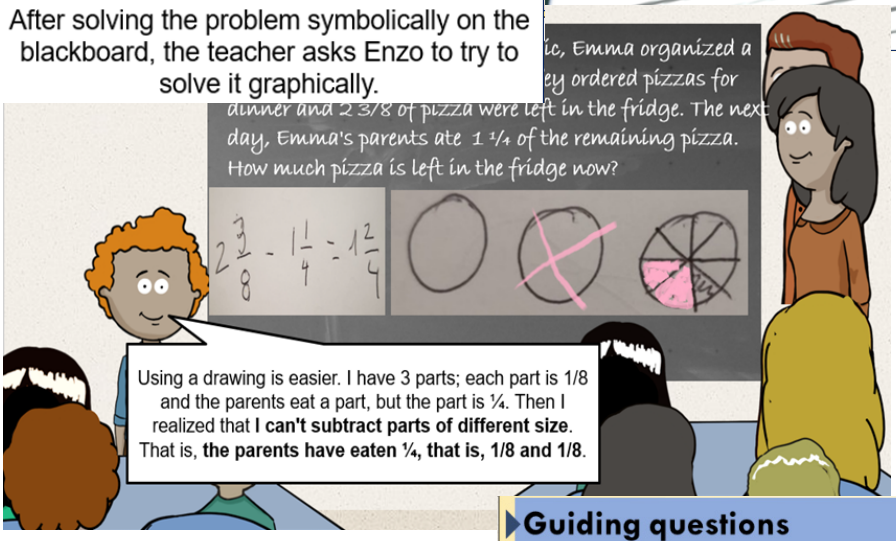
Figure 4.6 shows a sample vignette which is part of a course concept of Fractional Thinking for pre-service primary school teachers (grades 1-6) to enhance teachers' noticing of teaching-learning situations with fractions. The specific learning goals are attending to, interpreting and making decisions about students' thinking related to fractions.
This vignette describes a teaching-learning situation based on students' thinking of fraction operations. Pre-service primary school teachers are expected to attend to primary students' fractional thinking, interpret and make decisions about this situation, and assess the teacher's classroom management. The vignette shows a pre-service teacher in practice who is involved in a teaching-learning situation related to problem-solving with fractions. The primary teacher poses a problem and asks the students to solve it. After giving time to the group to solve the problem, she asks Enzo (a primary school student) to share his answer with the class. Enzo' answer shows different features of the primary students' difficulties with operations with fractions. Following the primary school teacher tries to help Enzo by asking him to change the representation to solve the situation.
To analyse this teaching situation, the pre-service primary teachers have a theoretical document with information on a hypothetical learning trajectory of students' fractional thinking (adapted from Battista, 2012) and about how students' solution strategies may develop toward more sophisticated and efficient ways of reasoning over time (Gibbons et al., 2018). The vignette includes three guiding questions about attending to, interpreting, and deciding according to the situation. These guiding questions are:

- Identify characteristics of Enzo's understanding related to the fraction concept. Justify your answer.
- After Enzo's answer, if you were the teacher, what would you do next?
- Why do you think the teacher has chosen Enzo's answer to share with the whole class? What advantages does this choice provide? Do you agree with her choice?





After solving the problem symbolically on the blackboard, the teacher asks Enzo to try to solve it graphically.



Guiding questions

- Identify characteristics of Enzo's understanding related to the fraction concept. Justify your answer
- After Enzo's answer, if you were the teacher, what would you do next?
- Why do you think the teacher has chosen Enzo's answer to share with the whole class? What advantages does this choice provide? Do you agree with her choice?

Figure 4.6: Vignette "Enzo's answer"

The first question is based on attending to the situation, where the mathematical elements involved in Enzo's answer allow pre-service teachers to identify characteristics of his understanding of the fraction concept. In this case, Enzo's answer shows his difficulties in understanding that the parts of the fraction must be of the same size to join or disjoin. After interpreting Enzo's answer, the second question provides an opportunity for pre-service teachers to decide how to continue with the instruction connecting the modelling strategy used by Enzo in solving the task with the counting strategies and the strategies that use properties of operations of fractions.
Finally, the last questions are designed to provide opportunities for pre-service teachers to assess their classroom management and to express their beliefs regarding classroom management.

4.7 Hidden opportunities for learning and argumentation: A vignette on a word problem with data

Target group:	Mathematics teachers at different level of experience
Profession-related learning objectives:	- Noticing related to learning support with respect of tasks based on statistical data - Awareness of learning opportunities related to statistical variation - Awareness of learning opportunities related to data-based argumentation
Topic:	Word problems, Diagrams with statistical data

The word problem given in this vignette refers to a set of data given in a diagram. The data is marked by statistical variation, which may be relevant for solving the task. However, the students and the teacher rather do not use or draw on the related learning potentials, which makes the vignette rich for reflection and the discussion of alternative teacher reactions. The specific possibilities and learning opportunities related to data-based argumentation may emerge from such a reflection with groups of mathematics teachers at different level of experience, beyond analysing the vignette students' thinking and the teacher's reactions.

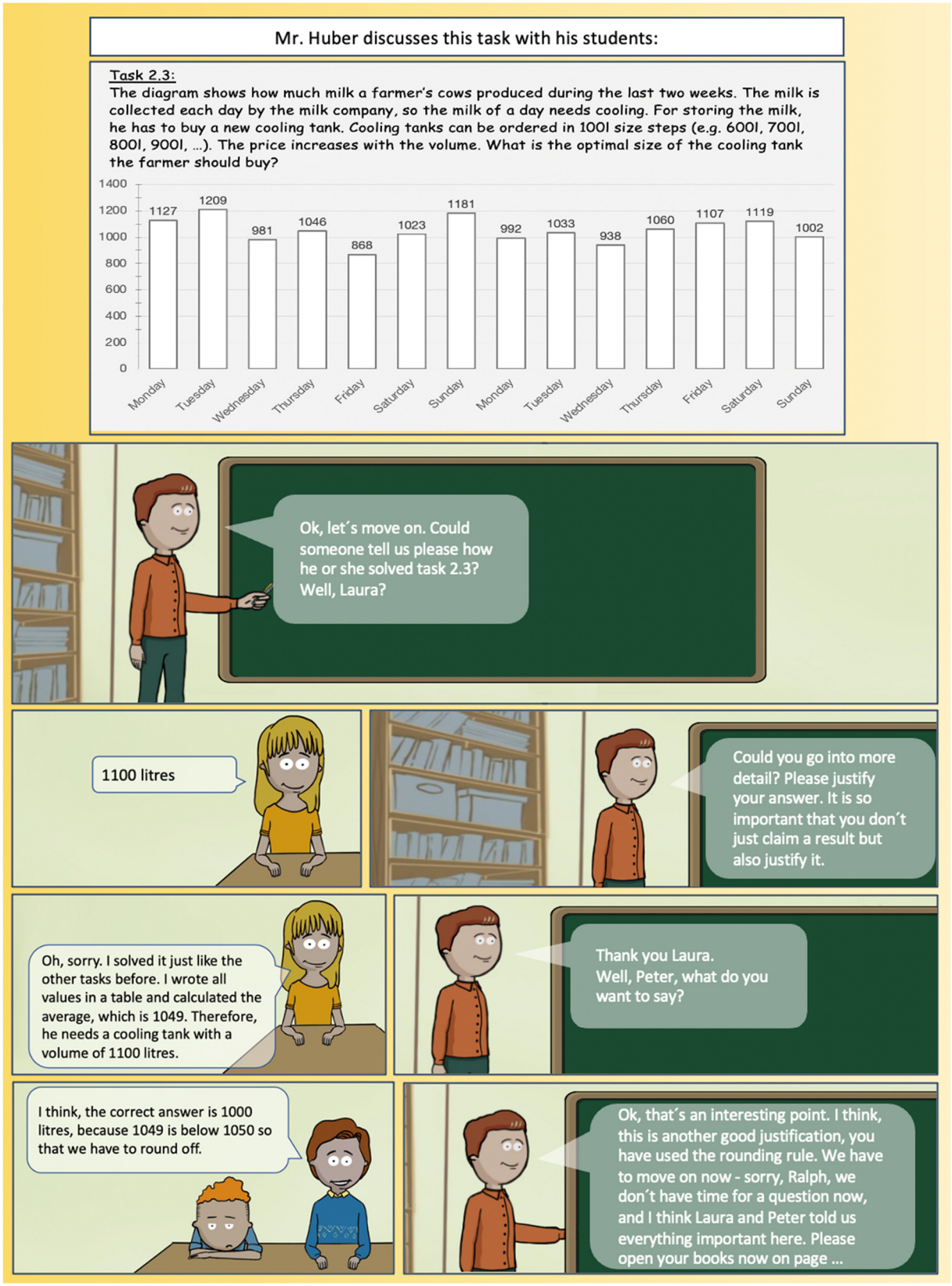


Figure 4.7: Vignette on a word problem with data

5. Reflecting on learning with vignettes and on the theory - practice relationship

The vignettes presented in the previous section refer to elements of mathematics education theory, in various ways. This activity follows up on the activity in section 3, it aims at reflecting on the correspondence between situation contexts represented in vignettes and relevant theory elements

Think of a theory element in mathematics education you are currently working on.

What learning goals for pre-service or in-service teachers would you associate with this theory element?

Think of the meaning of this learning goal or theory element for classroom practice and/or for a teacher's analysis of specific classroom situations. Try to describe a classroom situation or even types of classroom situations, in which the learning goal or theory element makes a difference, and explain how the learning goal or theory element matters for analysing/continuing/ etc. the classroom situation.

How can vignettes be designed …

- … to represent a particular content or aspect of teaching expertise related to this theory element?

- … to facilitate pre-service teachers' corresponding professional learning?

6. Using vignettes for (evaluation) research

Beyond providing stimuli for the profession-related learning of mathematics teachers, vignettes can also be used in research methodologies, in order to gather evidence on aspects of teachers' expertise, or profession-related learning outcomes, for instance. If vignettes are used as an element of a research design, they should be aligned with and consistently framed by the corresponding core elements of the respective study approach, such as its theoretical background and its research interest. In Buchbinder & Kuntze (2018), such aspects are discussed and a variety of examples of vignette-based studies is presented.

6.1 Introduction

In this section we focus on vignettes as a method that is both robust in quality for use in a practical way (e.g. for stimulus in a pre-service teacher course or as professional development with in-service teachers), and credible as a method for research purposes. In both cases the development of the vignette should be robust and credible so that it: fulfils its purpose by resonating with users; and is useful for eliciting information or provoking discussions and/or reflection as required.

To improve robustness and credibility it is important that there is consistency and alignment between the phenomena at the centre of the vignette, and the theoretical frameworks that underpin relevant constructs of the phenomena and the research design itself (Corbin and Strauss, 2008; Yilmaz, 2013). Not only should this overarching alignment exist but it is also suggested that construct validity is addressed when the vignette is constructed in this way (Skilling & Stylianides, 2020). In addition, context validity is addressed when the vignette material that portrays the hypothetical situations is fairly reflected and the users of the vignette resonate with it (Skilling & Stylianides, in press; see also section 9 of this booklet).

6.2 An example: Vignette-based research on teachers' beliefs

As a specific example, we discuss different vignettes that explore teachers' beliefs. Vignettes are particularly useful for probing and eliciting value-laden

constructs such as beliefs, understandings, judgements, attitudes, values, and complex thoughts (Jeffries & Maeder, 2005; Torres, 2009). When seeking information from individuals about their beliefs, understandings, attitudes etc. we assume that knowledge is constructed and understood from the individual's perspective. A vignette can be constructed to represent certain phenomena or situations from which questions can be asked to capture the beliefs or the meaning individuals ascribe to it. By capturing the beliefs, attitudes, values, norms and so on of several individuals, there are opportunities for explaining differences and similarities between them.

In education, seeking beliefs, understandings, judgements, attitudes, and values are relevant in how they shape teachers' instructional approaches. This includes the decisions made when planning lessons, for choosing tasks, mode of delivery, asking of questions, seeking feedback, and assessing student learning. Vignettes can be constructed and used with pre- and in-service teachers to probe and elicit their beliefs about the decisions they make in respect of their instructional approach. Eliciting such beliefs provides opportunities for personal and/or group reflection and discussion about what is noticed and analysed which are hallmarks of developing expertise and professional learning (also see sections 1.1-1.3).

6.3 An example of constructing two contrasting vignettes for researching in-service teacher beliefs about a certain phenomenon

A paper by Skilling & Stylianides (2020) argued that when using vignettes as a method in educational research, they should be constructed in ways that align with the research design and the theoretical paradigms. To help with the construction process a Vignette Framework using key characteristics and sub-characteristics to support the robustness of the vignette was proposed (Skilling and Stylianides, 2020). In Figure 6.1 below is an example of a pair of written vignettes that were constructed according to the framework.

The two written vignettes reflect a test preparation situation by Teacher A and Teacher B. The construction of the vignettes adhered to the three main elements of the Vignette Framework which attended to its conception, design, and administration (Skilling & Stylianides, 2020) of which more detailed information can be found in Chapter 11.

Teacher Vignettes

Context

Two Year 7 mathematics classes at one school will complete a topic test during second term. The teachers of each class provided students with information a week before the test about ways they could prepare. Below are suggestions by Teacher A and Teacher B. Please read each vignette and respond to the questions at the end. The line numbers for each vignette have been included to help make references to the text.

Teacher A

1. Teacher A reminded students about the upcoming topic test and handed out
2. a sheet with an outline of the key concepts that would likely be covered in
3. the test. The teacher suggested that the students set aside time for revision
4. and to make sure they practised each concept, by looking over their notebooks
5. as it was important for them to achieve a high grade on the test. The teacher
6. also mentioned that the students should ask questions in class if they were
7. unsure of the steps to solve questions. Alternatively, they could come and see
8. the teacher during break time to clarify any questions before the assessment.
9. In each lesson before the test the teacher set five practice questions in case
10. students had not been revising at home and students who got three or less
11. correct were advised they needed to study more.

Teacher B

12. Teacher B also reminded students about their upcoming topic test. The
13. students were asked to look through their mathematics notebooks and
14. textbooks during the lesson and recall specific topic concepts that they
15. thought would likely to be included in the test. Based on their class
16. work, the students were then asked to record how competent they felt
17. about each concept. During the lesson, the teacher also asked the
18. students to draw on their self-assessment notes and contribute to the
19. creation of a 'class' revision list, from which examples could be revised
20. during lessons before the test. The teacher also told the class that it
21. was expected that each student would develop individual revision
22. plans. Students would work on their individual plans at home, making
23. time to focus on mastering the concepts they believed they needed to
24. improve on. Throughout the week the teacher checked the revision
25. plans of each student and asked how they felt about their preparations.

Figure 6.1: Cognitive engagement vignette (Skilling & Stylianides, 2020, p. 16)

Overall, the conception and design of the Cognitive Engagement vignettes (Figure 6.1) support a connection from ontological and epistemological foundations to the vignette narratives, and the embedded constructs related to cognitive engagement (self-regulation strategies and metacognitive processes) in a 'typical' early secondary classroom situation in the UK. The words which are highlighted in green and blue in the Teacher A and Teacher B vignettes (Figure 6.1) reflect particular phases and subphases of self-regulation strategies (e.g. planning, strategy choice and use) and metacognitive processes (monitoring and reflecting), although they vary to the degree in which the vignette teacher promotes these. It is argued that by embedding these constructs deliberately within the vignette narrative assists with establishing construct validity and its resonance with

respondents supports content and context validity (Cohen et al., 2017; Skilling & Stylianides, 2020).

To accompany the vignette, eight open-ended questions were designed to elicit the beliefs and reported practices of the participants (in-service secondary mathematics teachers). Forming the questions in alignment with the vignettes assisted with the approach to coding the participants' written responses. For example, the participants were asked what they believed were similar and different between the two vignettes, and what were the most important and not important aspects. They were also asked which of the fictitious teachers they identified with, if they had used other approaches and in what circumstances. The nature of the study was qualitative, using open ended questions (with 40 participants) and in-depth interviews (with 17 of the 40 participants). The coding of the open-ended surveys with some descriptive statistics along with more details of the study can be found in Skilling & Stylianides (in press).

Open-ended questions:

1. What are some key similarities you perceive between the two teacher scenarios?
2. What are some key differences you perceive between the two teacher scenarios?
3. List up to two things that each of the teachers did that you believe were important for supporting the students' preparation for the test.
4. List up to two things that each of the teachers did that you **do not** believe were important for supporting students' test preparation.
5. Have you used any of the approaches used by Teacher A or B? If so, please list them.
6. Overall, which teacher do you identify with the most and why?
7. Can you name any other approaches that are not mentioned in the scenarios that you use to promote homework, study or revision (either during lessons or outside of the classroom)?
8. Choose one of the approaches that you mentioned in the previous question or any approach used by Teacher A or Teacher B that you consider important. Would you use or apply this approach for different groups of students? (e.g., Year levels, set or mixed ability classes, girls or boys). Please explain and provide an example if appropriate.

Figure 6.2: Cognitive engagement vignette questions (Skilling & Stylianides, 2020, p. 16)

6.4 An example of constructing one vignette for different research purposes with pre-service teachers

To illustrate the potential of vignettes in educational research in a broader variety of contexts, the following shows how the same vignettes can be employed as a research tool in different research studies. In particular, we present how vignettes accompanied by indicative questions or questionnaires can be applied to investigate noticing skills as well as beliefs, in qualitative as well as quantitative or mixed-method research studies. In

Figure 6.3, we introduce an illustrative sample vignette and four different research designs in which the vignette can be used as a key component of the data collection instrument. All four designs are intended for researching pre-service teachers, however, similar methodologies might also be applied with in-service teachers.

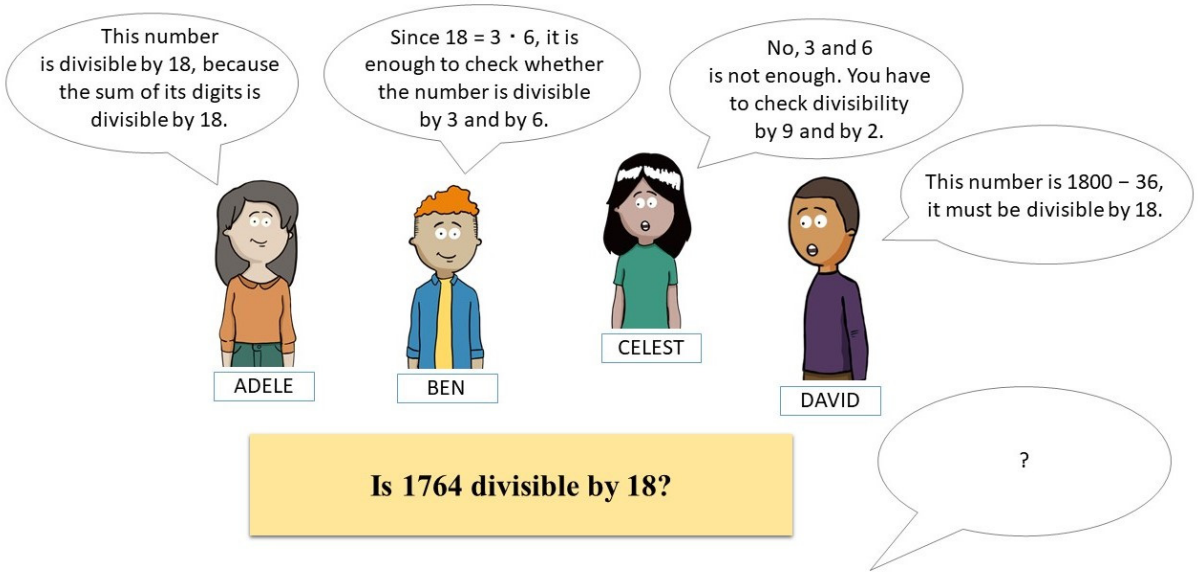


Figure 6.3: A vignette on divisibility by 18

The vignette in Figure 6.3 shows various opinions on the task assigned in the yellow frame; two of the opinions are correct (Celest, David), two of them are incorrect (Adele, Ben). Still, in all four statements, the argumentation of the students can be completed and/or improved. For primary school teachers, the mathematical content of the vignette (divisibility by 18) can be labelled as school-related content. For secondary school teachers, it belongs to the content taught in the school context. Consequently, the vignette can be used with pre-service and in-service primary as well as secondary school teachers.

The following overview introduces four different research designs (No. 1 to 4) in which the vignette in Figure 6.3 can be employed as a part of the data collection instrument. Two of the designs are qualitative (No. 1 and 3), one is mixed-methods (No. 2) and one quantitative (No. 4). For data collection, three of the designs use vignettes accompanied by one or more indicative questions (No. 1 to 3), the fourth design employs vignettes accompanied by a multiple choice questionnaire. Two of the designs focus on analysing aspects of pedagogical content knowledge (No. 1 and 2), two on beliefs (No. 3 and 4). We have already conducted research studies implementing the

four designs, the references are included in the respective descriptions below. The studies based on designs No. 1 and 2 are closely related, they not only have the same focus but also the same data collection instrument (a set of two vignettes accompanied by the same indicative question). The first three studies (designs No. 1 to 3) included specifically the vignette in Figure 6.3, the fourth study (design No. 4) was carried out with a vignette in a similar format. However, the methodology of the fourth study could be repeated with the vignette in Figure 6.3 without any further modifications.

Design No. 1 – Qualitative

- Focus: noticing as helping children to learn, adaptive individual learning support, content knowledge
- Research tool: two vignettes assigned to pre-service teachers with an indicative question: “How could you help the student teachers (1) to correct their answers or (2) to improve their argumentation?”
- Research question: “What kind of content knowledge can be revealed in pre-service teachers when using Concept Cartoons as a written assessment tool within a mathematics content course?”
- For more details see Samková (2022)

Design No. 2 – Mixed Methods

- Focus: noticing as helping children to learn, adaptive individual learning support, content knowledge
- Research tool: two vignettes assigned to pre-service teachers with an indicative question: “How could you help the student teachers (1) to correct their answers or (2) to improve their argumentation?”
- Research questions:
    (1) “To what extent are pre-service teachers able to provide learning support in a vignette-based setting showing a fictitious situation in the content area of divisibility?”
    (2) “What role does their content knowledge (CK) play in this context?”
    (3) “In which form do they suggest to provide learning support and is it possible to infer to their conceptions of “helping learners” from the findings?”
- For more details see Kuntze et al. (2022)

Design No. 3 – Qualitative

- Focus: beliefs about vignettes and their use
- Research tool: an open question assigned to pre-service teachers after their work with two vignettes – “What do you think: How can

working with Concept Cartoons help you to improve your learning related to divisibility?"
- Research question: "How do pre-service teachers reflect on their work with Concept Cartoons representing school-related mathematics content?"
- For more details see Samková & Friesen (2022)

Design No. 4 – Quantitative
- Focus: beliefs about vignettes and their use
- Research tool: a questionnaire comprising of 7 indicator items on views of pre-service teachers after their work with vignettes, the items addressing pre-service teachers' perceived motivation, authenticity and immersion when working with the cartoons, evaluating to what extent the participants felt supported by the use of cartoons in their learning about problem solving, and evaluating the perceived potential of cartoons for teacher education in a more general way
- Research question: "How do the participating pre-service teachers perceive the potential of cartoon vignettes in terms of their professional learning?"
- For more details see Friesen & Knox (2022); for a similar study based on a 20-item multiple choice questionnaire see Bernabeu et al. (2022)

6.5 Conclusion and outlook

In this book section, spotlights on vignette-based research approaches have been given – the still dynamically evolving field of possible uses of vignettes in research methodologies is of course much broader (cf. e.g. Buchbinder & Kuntze, 2018) and promises multifaceted and rich developments of further approaches in the future. In Friesen (2017, see also e.g. Friesen & Kuntze, 2016) for instance, vignettes in video, cartoon, and text format are used as items of a competence measurement instrument, which can be used to reconstruct a hierarchical construct (the competence of analysing how representations are being dealt with in classroom situations) through a Rasch modelling of the test data. This study is particularly interesting, as the very situated and case-based vignette approach spans towards a situation-overarching competence construct. The corresponding instrument can thus also be used to evaluate teacher education courses by monitoring the

learners’ progress quantitatively on the corresponding competence dimension.

7. A digital tool for representing classroom situations: The DIVER CREATE tool

In this section of the booklet, the digital DIVER CREATE Tool ("Developing and Investigating Vignettes in Teacher Education and Research") is described, which supports in particular the representation of classroom situations in cartoon format (see Figure 7.1 below for first examples).

7.1 The DIVER tool - Designing and Investigating Vignettes for Teacher Education and Research

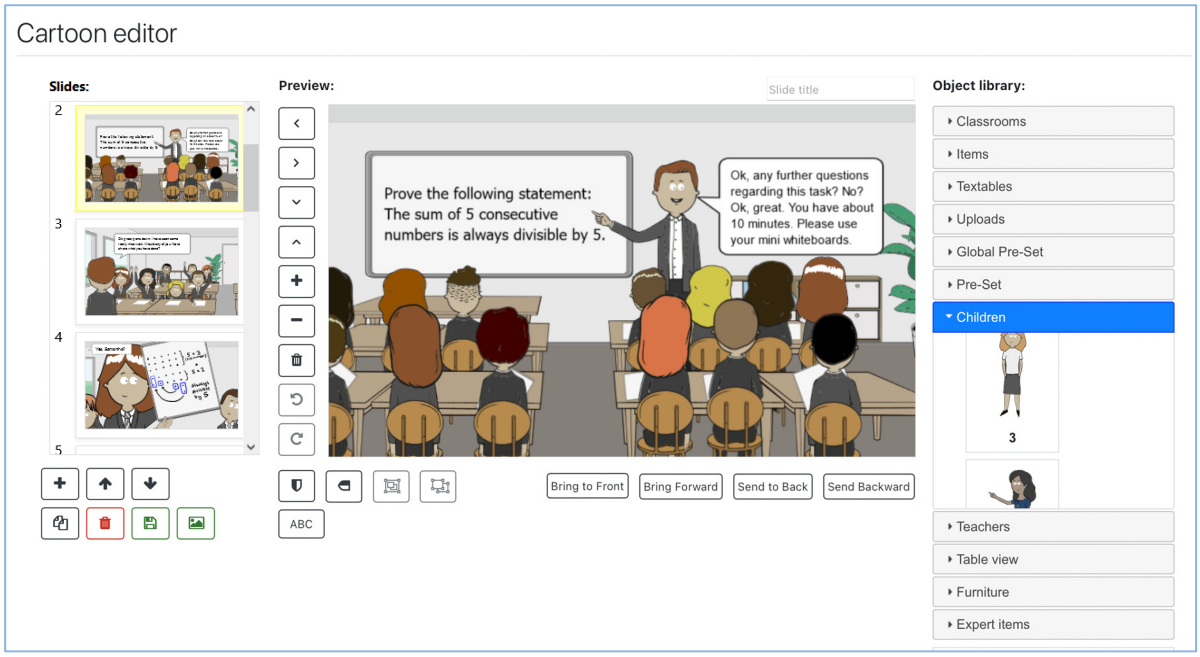
As cartoon-based vignettes can be equally suitable to elicit teachers' analysing compared to text and video vignettes (e.g. Friesen & Kuntze, 2016a; Herbst et al., 2011; Friesen, 2017), the potential of cartoon vignettes has gained increased attention. Cartoon vignettes combine numerous advantages ascribed to video and text, namely among others:

- Possibility of a systematic, theory-based design and variation of classroom situations
- individual characteristics that are important to comprehend a classroom situation can easily be added

(Herbst et al., 2011; Friesen & Kuntze, 2018; Ivars et al., 2020)

DIVER is based the learning platform Moodle for creating cartoon-based vignettes, is multi-lingual (English, Spanish, Czech and German), and facilitates the creation of cartoon vignettes for digital learning environments and research instruments. Furthermore, it

- enables the collaborative reflection and exchange of vignettes
- can be used to draw out aspects of teacher expertise, teacher beliefs and convictions, teacher noticing and other aspects of teacher expertise
- can describe mathematical details relevant for content knowledge, instructional knowledge and student thinking
- can encourage reflection on teacher decision making, among further aspects.



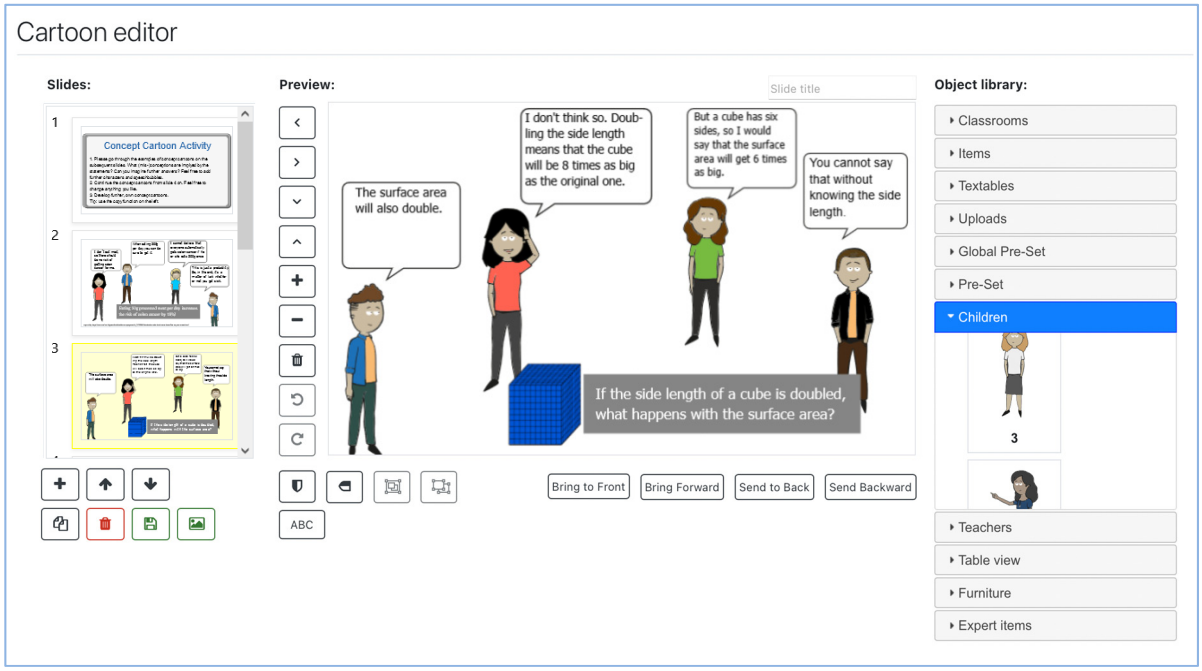


Figure 7.1. Two examples of classroom scenarios in vignettes created with the DIVER tool

7.2 Context of the DIVER tool

Providing teachers with the opportunity to engage in representations of practice has proven to be an effective approach in teacher education and corresponding research (Herbst et al., 2011). Questions regarding possible designs of vignette-based learning and testing environments have become essential in this context and the potential of cartoon-based vignettes has gained increased attention (Friesen & Kuntze, 2018). Cartoon vignettes combine numerous advantages ascribed to video and text vignettes: Cartoons allow the systematic, theory-based design and variation of classroom situations whereas individual characteristics that are important to comprehend a classroom situation can easily be added (Friesen & Kuntze, 2018; Herbst et al., 2011). Moreover, cartoon-based vignettes are equally suitable to develop and assess teachers' competence in analysing classroom situations, compared to the vignette formats video and text (Friesen, 2017). In the framework of the project coReflect@maths, DIVER (Developing and Investigating Vignettes in Teacher Education and Research), a digital tool for creating cartoon-based vignettes, has been developed and programmed (Ivars et al., 2020). The DIVER tool does not only allow the creation of cartoon vignettes (e.g., by arranging student and teacher characters in classroom environments, adding speech bubbles, etc.), but can also enhance the collaborative reflection and exchange of vignettes within the learning platform Moodle. The DIVER-Tool aims thus at engaging Mathematics pre-service and in-service teachers in active learning by analysing and creating representations of Mathematics classroom practice. Moreover, cartoon-based vignettes can be related to numerous theoretical frameworks and aspects of quality teaching in the Mathematics classroom, e.g., the use of multiple representations, the productive handling of mistakes, the noticing of students' geometrical thinking, etc.

7.3 What is the DIVER tool?

DIVER is the abbreviation for "Developing and Investigating Vignettes in Teacher Education and Research". The DIVER tool is a digital tool for creating and exchanging cartoon-based vignettes. DIVER allows the fast and easy digital development of cartoon-based vignettes (e.g., by putting together different student and teacher characters, adding speech bubbles, uploading photos of students' work, etc.) with a highly intuitive user interface. DIVER has been developed as plug-in for the learning platform Moodle, which allows, e.g., developing and storing cartoon vignettes

directly within the learning platform or setting tasks and activities together with the vignettes (e.g., for analysing classroom scenarios or finishing / continuing a given classroom situation). The user management and organisation of courses is facilitated by the learning platform Moodle which also provides data protection and the connectivity to already existing Moodle-based features and activities (e.g., submit a task). No installation is necessary for the use of DIVER since it can be accessed via standard web browsers.

7.4 Who are the main target groups of the DIVER tool?

The main target groups of DIVER are teacher educators and student teachers. Teacher educators can, e.g., develop and implement vignette-based learning activities in their courses for pre-service teachers or in professional development programmes for in-service teachers. Cartoon vignettes can also be used for evaluating and reflecting professional growth or for formative assessment. Student teachers can create and analyse vignettes as activity in university courses or for documenting and reflecting classroom events observed in practical teaching experiences. Since DIVER is available in the languages English, Spanish, Czech and German, it can be used in different countries and associated cultural contexts and helps to reduce language barriers that are often involved in the use of digital tools.

7.5 What are the features and functions of the DIVER tool?

This section presents an overview of the features and functionalities of the DIVER tool. In the following Figures 7.2 and 7.3, screenshots are given on the base of which the features and functions will be described in the following.

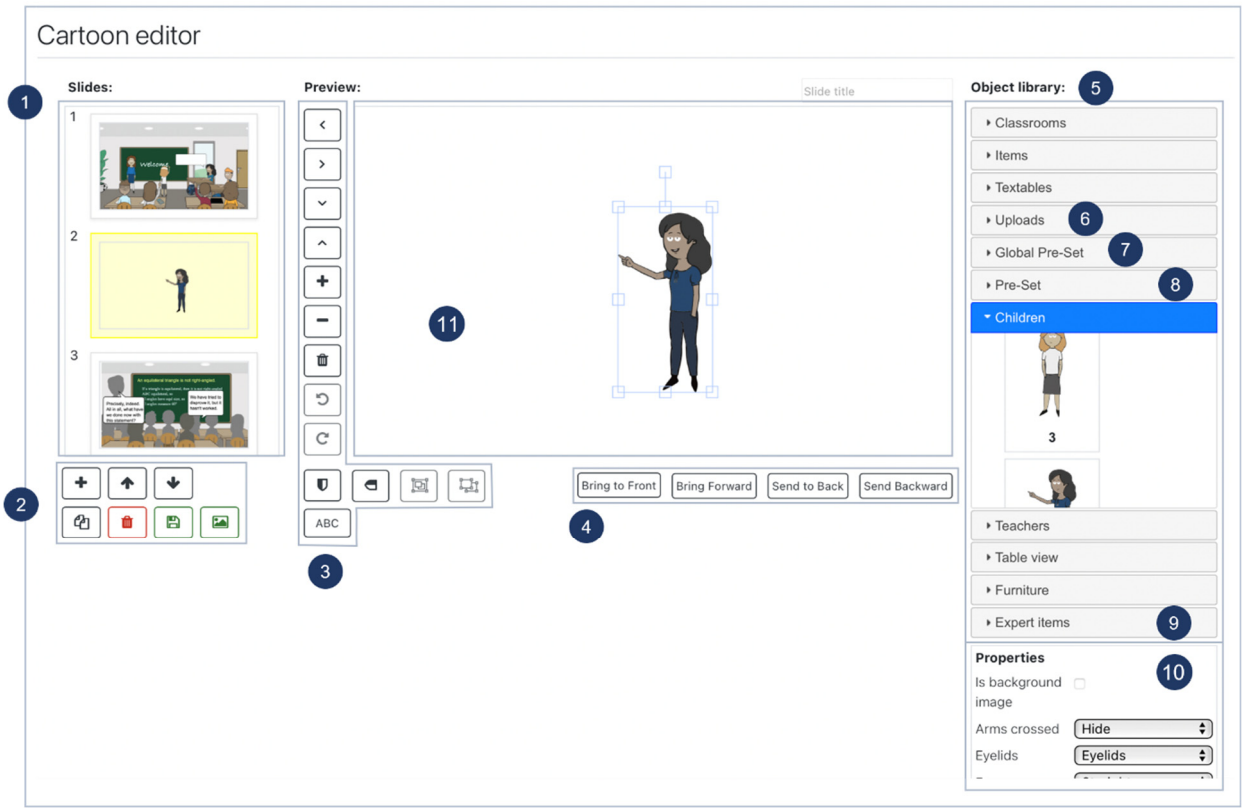


Figure 7.2. Screenshot of the DIVER CREATE cartoon editor window

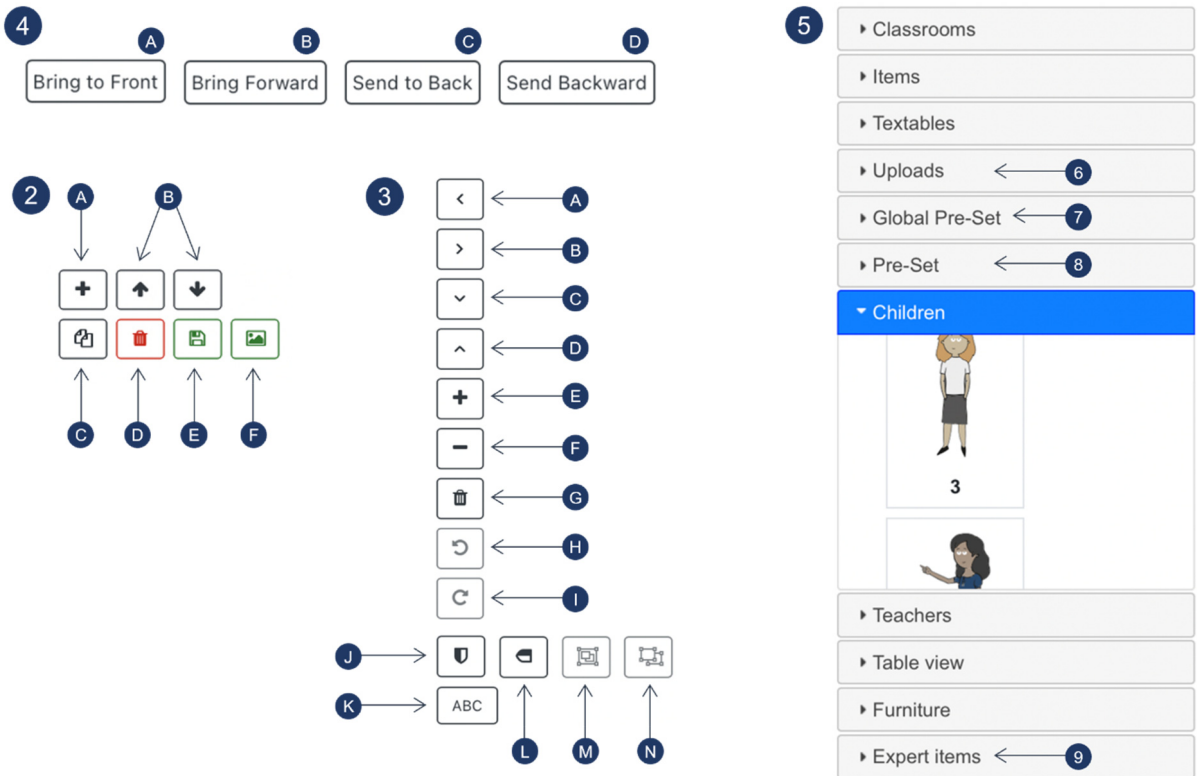


Figure 7.3. Overview of functionality elements in DIVER CREATE

7.5.1 Slides overview:

The slides are numbered and presented in a scrollable menu (left side in Figure 7.2). When a slide is selected, it appears in the editing window in the centre and can be edited.

7.5.2 Buttons for editing slides (Figure 7.3, number 2):

A) Add a new (blank) slide
B) Move selected slide up or down
C) Duplicate selected slide
D) Delete selected slide
E) Save file
F) Create a pre-set consisting of all objects/groups implemented in the selected slide. Generated pre-sets are available in the “pre-set” section (8) in the object library.

7.5.3 Buttons for editing objects and object groups (Figure 7.3, number 3):

A) Move selected object/object group left
B) Move selected object/object group right
C) Move selected object/object group down
D) Move selected object/object group up
E) Zoom in
F) Zoom out
G) Delete selected object/object group
H) Rotate selected object/object group counterclockwise
I) Rotate selected object/object group clockwise
J) Flip selected object/object group horizontally
K) Insert text
L) Flip selected object/object group vertically
M) Group selected objects
N) Ungroup selected object group

7.5.4 Buttons for arranging overlapping objects/object groups (Figure 7.3, number 4):

A) Bring to front: send selected object/object group to front
B) Bring forwards: send selected object/object group one layer further forwards
C) Bring to back: send selected object/object group to back

D) Send backwards: send selected object/object group one layer further backwards

7.5.5 *Object Library*

The object library comprises several graphical elements in different categories (see Figure 7.4). A graphical element can be implemented in a cartoon slide by clicking on it. Further features of the object library are explained in the following.

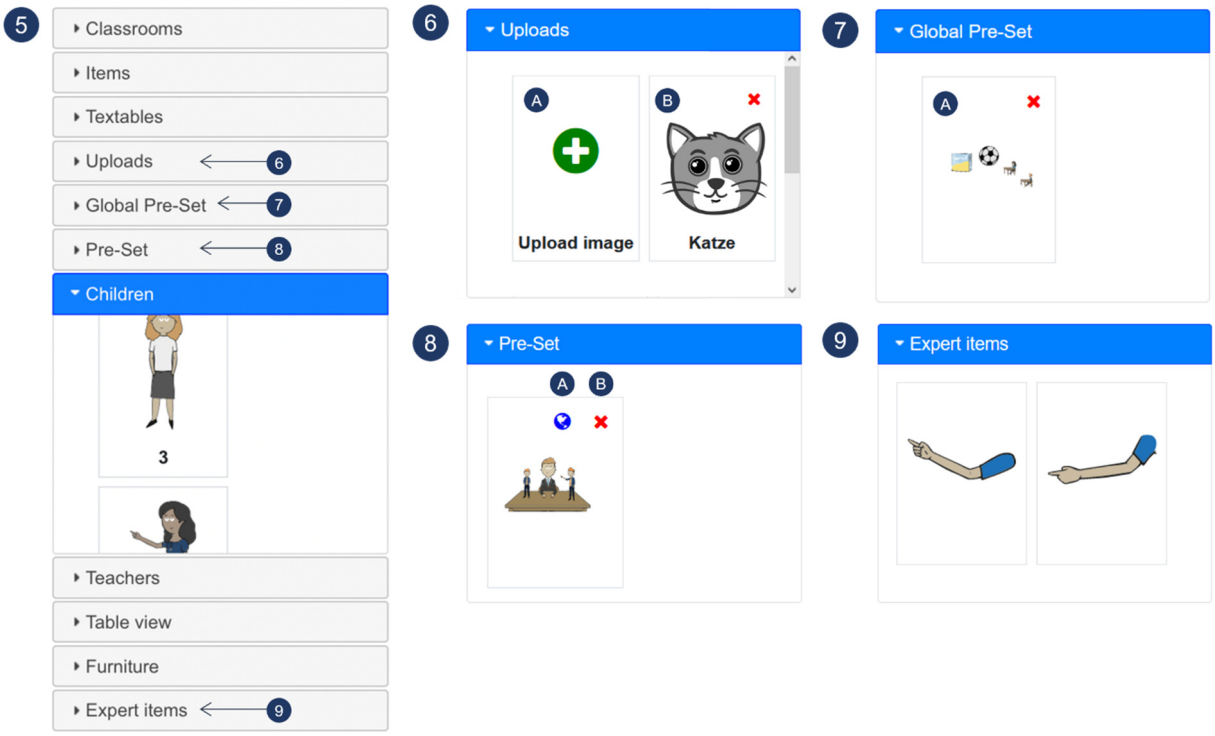


Figure 7.4. Graphical elements from the object library

7.5.6 *Uploads (Figures 7.4 and 7.2, number 6)*

A) By clicking the upload button, you can upload an image in .png and .jpeg format. The image can then be used in cartoons (on a personal level).

B) Example of an uploaded image

7.5.7 *Global Pre-Set (Figures 7.4 and 7.2, number 7)*

The global pre-set section provides several pre-configured scenarios, which can flexibly be added to cartoon slides. The global pre-sets are available for all DIVER users on the respective Moodle system.

A) Example of a global pre-set ready to be used in a cartoon.

7.5.8 *Pre-Set (Figures 7.4 and 7.2, number 8)*

The pre-set button (2) allows to save all graphical elements (including their properties, scaling, position, etc.) of a slide as a pre-set, which are available on a personal level and can be implemented flexibly into other cartoon slides.

A) By clicking the globe (available only for authorised users), the pre-set can be defined as a global pre-set available for all users.

B) By clicking the red cross, the pre-set can be deleted.

7.5.9 *Expert items (Figures 7.4 and 7.2, number 9)*

The expert items provide some specific features with high flexibility for depicting classroom situations. For instance, you can use the graphical elements in this category for implementing an arm in a certain angle, which is not possible with the standard properties.

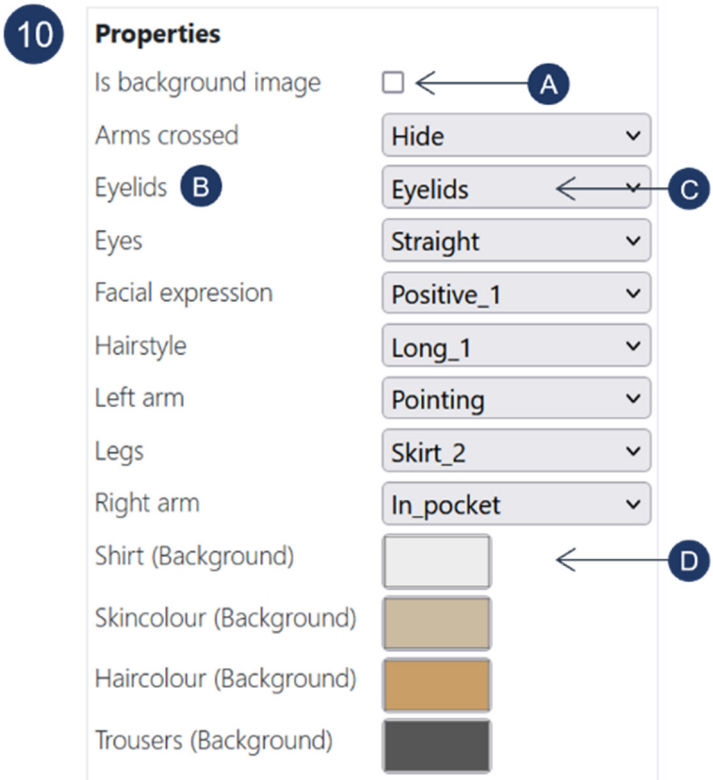


Figure 7.5. Properties menu related to graphical elements in DIVER CREATE

7.5.10 *Properties (Figures 7.5 and 7.2, number 10)*

Properties (such as arm positions, hairstyles, colours) are changeable specifications of a selected graphical element and they can be adapted in various ways:

A) Clicking this box defines the selected graphic as background image. Only one graphic can be set as background image.

B) “Eyelids” is an example of a property, which can be defined for characters.
C) Allows to activate/deactivate the visibility of the eyelids of the selected graphic. Other examples for this type of properties are arm positions or hairstyles.
D) Allows to change the colour of certain parts of the graphic (shirt). Depending on the operating system and browser, a pipette function is available.

7.5.11 Working area (Figure 7.2, number 11)

Graphics can be arranged here.

7.6 What else is important for working with the DIVER tool?

Essential guidelines for basic use:

- DIVER works best with the browsers Chrome, Edge and Safari.
- Save slides frequently when implementing changes - this helps to avoid problems, as DIVER is a browser-based software.
- **DIVER does not stop loading?** As DIVER is a browser-based software, this may happen from time to time, e.g. due to problems with the internet connection. In this case, press the **refresh button** of your browser. If this does not solve the problem, please check your internet connection.
- **You can’t edit properties of a graphic?** If your graphic is in a group, you need to ungroup the group and select the graphic individually to be able to edit its properties. Also, some graphics don’t have properties at all. Especially in the category “Items”.

Advanced guidelines for expert users:

- **Using the “Global pre-sets”:** In the category “Global pre-sets” you find several pre-sets with scenarios frequently occurring in classrooms. You can use these pre-sets as a starting point for developing your individual cartoon slides. If you have the required permissions, you can define personal pre-sets (see below) as global pre-sets available for all users on your Moodle platform.
- **Using the pre-set button:** The pre-set button provides a very flexible clipboard function. When pressing the pre-set button, all graphical elements of a slide appear as a (personal) pre-set in the object library.

You can insert these stored elements in all of your slides. As you can use your pre-sets in all of your DIVER cartoons, this also allows to copy slides from one cartoon to another.

- **Using the upload function:** Do you need additional graphical elements? The upload function allows to implement further graphical elements in the formats .png and .jpg. We recommend using the .png format, as it supports transparent backgrounds. You can use the upload function also to upload formulas or text you created in another programme. For this, you have to save the text or formula in the format .png with a transparent background.
- **Using expert items:** The expert items provide some specific features which provide high flexibility for realising classroom situations. For instance, you can use the graphical elements in this category for implementing an arm in a certain angle, which is not possible with the standard properties.
- **Creating "shadow vignettes"** (Figure 7.6): Some contexts may require a particular sensitivity regarding diversity or the need to fade individual characteristics of the depicted characters. For this purpose, so-called *shadow characters* can be a solution. To create a shadow character, set all colours of the cartoon characters, including the outlines, to one and the same grey tone, and hide all elements related to the facial expression (see example below).

Figure 7.6. Example for a cartoon representation with "shadow" persons

7.7 Where can I read more about DIVER and cartoon vignettes?

Further resources related to DIVER and cartoon vignettes can be found in Krummenauer et al. (2020), Ivars et al. (2020), Friesen (2017), Friesen & Kuntze (2018), and Herbst et al. (2011), among other references cited also throughout this booklet.

8. Digitally supported vignette design

Consider how you might use DIVER for creating vignettes.
Which different possibilities of creating a cartoon vignette out of an imagined classroom situation can you think of? Develop variants of your vignette and think of how you would implement it using the DIVER CREATE tool.

Observe your working process: Which working strategies do you prefer when creating cartoon vignettes? Do you observe phases in the process of creating the vignette? In which phases can the DIVER CREATE tool support play a role?

9. Towards interconnected case-based learning opportunities: Vignette-based course concepts

Vignettes can be stand-alone learning opportunities in mathematics teacher education, for introducing topics, promoting discussion and reflection on classrooms among pre-service teachers or in groups of practicing teachers - however, vignette-based work can also be interconnected in networks of case-based learning opportunities. If case-based learning is acknowledged as an overarching principle in teacher education (e.g. Friesen & Kuntze, 2020), then the pathway is open towards larger course structures which connect vignettes as professional learning opportunities. In the project coReflect@maths, so-called *course concepts* have been developed and even been subject to empirical evaluation research.
In the following, we will present an overview of a series of examples of course concepts, in order to showcase how such networks of vignette-based learning opportunities can be designed and structured.

The course concepts reflect the idea of connecting case-based learning opportunities in networks which are designed according to specific case-overarching learning goals, such as the following, which have been developed and documented in the coReflect@maths project:

- *Enhancing teachers' noticing of students' mathematical thinking related to fractions*: A key target of this course is to foster the participants' noticing related to the mathematical thinking of the students as well as interpreting of their understanding. The development of theoretical knowledge on hypothetical learning trajectories is also in the foreground, so as to support decision-making for the mathematics classroom.

- *Developing pre-service teachers' professional knowledge and analysing of students' solving of (non-routine) word problems*: In the framework of this course, the analysis of solution approaches by students to non-routine-word problems is intended to be supported, as well as professional knowledge on the curricular embedding of problem solving on characteristics of problem solving tasks, strategies of learners, obstacles and possibilities of support.

- *Enhancing noticing of geometry teaching in primary education*: This course focuses on noticing in the geometry classroom, in particular the geometrical thinking of the students is in the centre, together with the analysis of task material and textbook pages, of teacher-student interactions and of task design.

- *Eliciting discussion on topics related to school practice in elementary mathematics – solving and assessing tasks that are open*: This course foregrounds objectives related to promoting the Discussion among pre-service teachers on key topics of classroom practice, in particular on solution processes and open tasks.

- *Eliciting discussion on topics related to school practice in elementary mathematics – Understanding fractions*: Aspects of solving tasks on fractions and their characteristics are in the focus of this course. The tasks in the vignettes are open and allow multiple solution pathways. PCK, namely knowledge about tasks (multiple solution pathways), about learners (multiple solution ideas) and about instruction (assessment with tasks) are intended to be fostered.

- *Dealing with multiple representations in the mathematics classroom*: This course aims at building up and promoting pre-service teachers' competence of analysing how representations of mathematical objects are dealt with, a flexible analysis focus is foregrounded, which combines the analysis of task material and textbook pages, the analysis of interaction and classroom dialogues, as well as the analysis of students' difficulties. Moreover, relevant professional knowledge, views and specific awareness is intended to be supported.

- *Dealing with heterogeneous learning prerequisites / diversity in the mathematics classroom*: This course concentrates on promoting competences related to analysing in the following areas: Analysing students‘ thinking, their learning prerequisites and difficulties, analysing tasks and contents related to possibilities to conceive of learning opportunities and learning support on various levels of complexity, as well as analysing adaptive possibilities of connecting with learning prerequisites and learning needs. Moreover, the participants are supported in their relevant professional knowledge, views and specific awareness (Kuntze & Friesen, 2018).

- *Using mistakes as learning opportunities in the mathematics classroom*: Building up and fostering competences of analysing in the

following domains is in the centre of interest: Mistakes (mathematical analysis), students‘ thinking "behind" the mistake, the learning potential related to the mistake (both for the student who has made the mistake and for all students in the classroom), dealing with mistakes in the classroom and different reaction possibilities in order to promote the students’ learning from the mistake. Moreover, relevant professional knowledge, views and specific awareness is intended to be supported.

- *Promoting argumentation in the mathematics classroom*: This course concept aims at building up and fostering competences of analysing mathematical contents in order to identify argumentation opportunities, analysing students’ argumentations, analysing classroom situations with respect of argumentation opportunities, as well as analysing to which extent different teacher reactions could promote students’ argumentation. Moreover, relevant professional knowledge, views and specific awareness is intended to be supported.

For all these course concepts, a set of characterising questions has been answered so as to describe their key design features and in order to facilitate a systematic comparison between different ways of implementing vignettes as case-based profession-related learning opportunities into larger frameworks, interconnecting them by larger goals of professional learning or specific curricular structures related to the profession of mathematics teachers.

The following characterising questions are answered for all course concepts:

- What is the target group of the course?
- What are the aims and the learning goals of the course?
- What is the related theory?
- How is the course structured?
- What does the course format look like? (organisation of sessions, online/offline/hybrid, duration, …)
- What is represented and in which format (video, text, cartoon or combination)?
- How many vignettes are part of the course?
- Are the vignettes found, authentic, adapted or scripted?
- Is there complementing text material for the course participants?

Furthermore, references to related literature is given and contact information related to the course concept is provided.

In the following, we give examples of such a course concept in the structured presentation format which was chosen to document all the above-mentioned courses on coreflect.eu (see Figure 9.1) and a documented vignette example from this example course concept (see Figure 9.2).

Figure 9.1. Overview of a course concept as provided on www.coreflect.eu

Figure 9.2. Overview of a documented vignette from www.coreflect.eu

All course concepts and documented vignettes can be downloaded from www.coreflect.eu or from the EU results platform (https://erasmus-plus.ec.europa.eu/projects)

10. Reflecting on sample course concepts

Choose a seminar topic from your recent pre-service teacher education modules: Can you think of introducing case-based elements in it, which make a direct connection to classroom practice?

Can you think of using a set of vignettes throughout the seminar? How could these be designed and interconnected? How are they connected with theoretical elements of the seminar?

Compare with the course concepts in the previous section: Can you use structural elements from the given examples or do you see a need of specific adaptation to your seminar topic?

An observation from the coReflect@maths project work is that methodological elements of learning with vignettes might impact back also on the curriculum of pre-service mathematics teacher education or in-service teacher professional development activities. Can you think of such an effect in the topic domain of your seminar or have you observed a shift in the learning goals when working on the questions above?

11. Deepening: Theory, Goals, Vignette validity

Vignettes are representations of practice which can at the same time reflect and represent theory and particular constructs (see sections 1.1-1.3 of this booklet). Vignette-based learning opportunities usually have goals according to which they might have been designed or they might be used as opportunities of reflection and criteria-based discussion. This results in a set of requirements related to vignette validity.

11.1 Aspects of vignette validity: practice validity, goal validity and theory validity

The notion of validity of a representation (something that stands for something else, cf. Goldin & Shteingold, 2001) is associated with the property of somehow being meaningful and consistent with respect to the object being represented. Vignettes, which are considered as representations of practice thus should be valid in the sense that they should be meaningful for a practice context or a set of practice contexts, such as, for example, a classroom situation or a type of classroom situations, a process such as how a typical situation might unfold potentially even for a situation-overarching pattern, e.g. in a teacher's communication practice in the mathematics classroom. Indeed, the groups of mathematics educators and researchers working as part of the coReflect@maths project who come from different settings (countries, cultures and social structures) still found many similarities between their approaches in respect of lesson events and teacher practices. This fostered many discussions about the differences and similarities between representation of practice and classroom situations between the group and also provided an interesting basis from which we could conduct comparative research – in fact, these discussions and this exchange were closely related to efforts of establishing and assuring validity of representations of classroom situations and related opportunities for professional learning. For example, using the same vignettes in different countries and courses, using vignettes developed in one setting and using it in a modified way in another setting, and comparing vignettes that have similar research aims such as eliciting pre-service teacher beliefs and understandings require considerations about the meaningfulness of a representation of practice for specific practice contexts.

Beyond these aspects of *practice validity*, other validity aspects result from the double representational role of vignettes, which also represent theory and the goals associated with a vignette-based learning opportunity (Fig. 11.1).

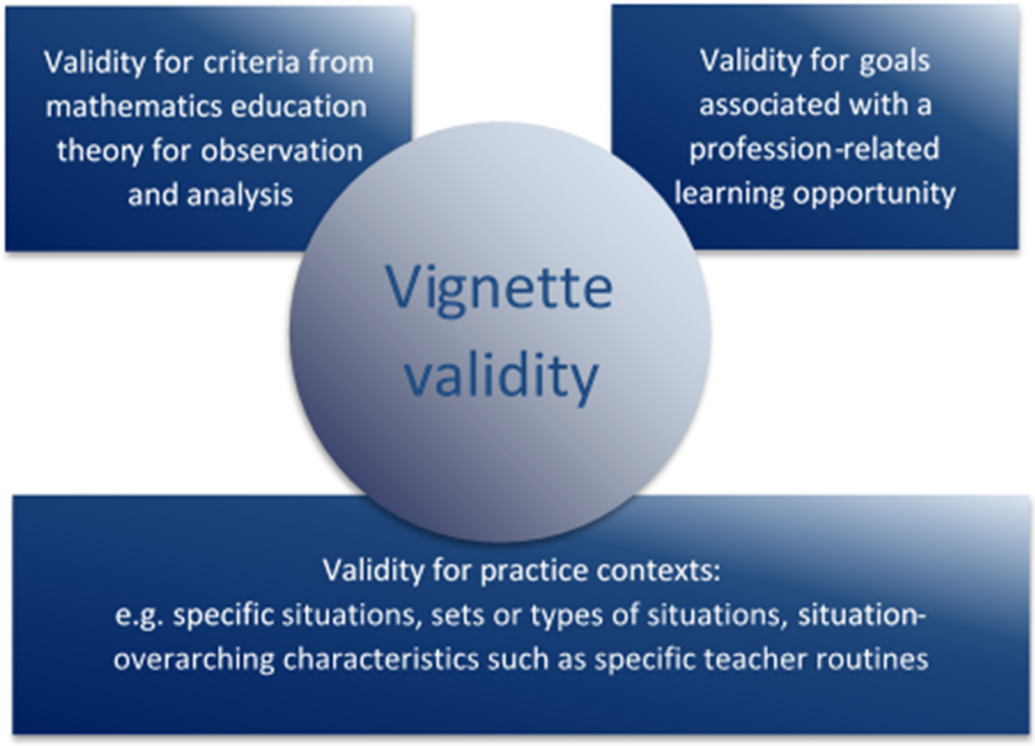


Figure 11.1: Aspects of vignette validity

Within a vignette-based professional learning opportunity, a vignette usually has been selected or even created according to a specific theoretically framed perspective. This means that in turn, the vignette should be valid for the theory-based construct(s) which have led to the choice of the vignette or its design (*theory validity*). Considering this aspect of validity is challenging (e.g. Kuntze & Friesen, 2016b; Skilling & Stylianides, 2020) as a representation of practice often shows a specific case of e.g. a classroom situation on the one side, and a theoretical construct from mathematics education on the other side, for instance a specific variable related to instructional quality (e.g. cognitive activation (e.g. Clausen, Reusser & Klieme, 2003), are mostly relatively distant from each other. Here, the almost antagonist role of cases on the one side and case-overarching theory elements on the other side has to be taken into account (Friesen & Kuntze, 2020).

On a meta-level, a vignette also stands for the *goal*(s) associated with its use in a profession-related learning opportunity (*goal validity*). If teachers, for example, should be stimulated to reflect on interaction patterns in the

mathematics classroom which are potentially detrimental for promoting students' argumentation, then the vignette somehow has to represent this professional development goal.
At the same time, the classroom situation shown in the vignette should be valid for real-life situations which happen in authentic classrooms or typical patterns of mathematics teachers' behaviour (*practice validity*). In these cases the vignettes are particularly helpful for stimulating reflections of in-service teachers, whereby they can raise issues that they have encountered in classrooms. At the same time, the vignette should be valid for theory-based observation and analysis criteria (*theory validity*, e.g. criteria related to theory-based discourse rules, cf. Cohors-Fresenborg & Kaune, 2001, 2003). Vignette validity is thus an interwoven and multi-perspective requirement.

11.2 Establishing the vignette method as valid and credible

As discussed in Chapter 6, vignettes can be used in professional learning settings (e.g. for stimulus in a pre-service teacher course and/or as professional development with in-service teachers) and also as a methodological element for research purposes. In both cases the development of the vignette should be robust and credible so that it: 1) fulfils its purpose for resonating with the users of it and 2) is useful for eliciting information or provoking discussions and reflection as required.
The nature of social science (and education) research includes a number of underlying assumptions that may influence the approach for seeking the *truth* about or investigating a particular phenomenon. Empirical research provides for a systematic and controlled way of carrying out such investigations that guide particular approaches and methods, not only to limit errors but to ensure that procedures and results are open to scrutiny (Cohen, et al., 2017). In Figure 11.2 the different kinds of assumptions that are generally agreed upon are represented by the central nested circles with explanation of these in the corresponding green boxes on the left hand side. The assumptions and position of the research affect the nature of the inquiry, relevant theoretical frameworks and constructs and methodological concerns. In essence these aspects of research design should be aligned with the underlying assumptions and aims of the research because this helps support its overall validity and credibility (and is represented by the grey box and connecting arrow on the right hand side of Figure 11.2).

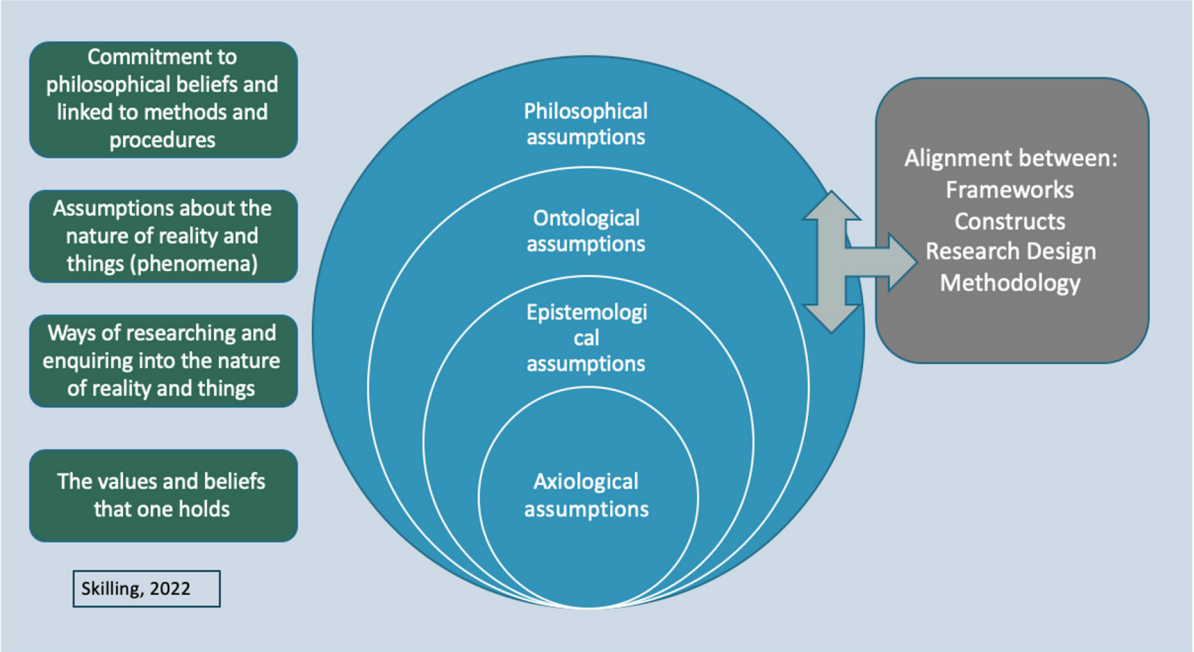


Figure 11.2 Methodological awareness and alignment in vignette-based research

When considering using vignettes as the sole or main methodological element in research it is imperative that attention be paid to both the purpose of the investigation and how the vignettes reflect the aims of the research as well as to their positioning within the research design (e.g. as an appropriate methodological tool). This dual focalisation is crucial for establishing the validity and credibility of the vignette itself, not to mention its desired effectiveness as a tool for the research.

To improve robustness and credibility it is important that there is consistency and alignment between the phenomena at the centre of the vignette, and the theoretical frameworks that underpin relevant constructs of the phenomena and the research design itself (Corbin & Strauss, 2008; Yilmaz, 2013). This overarching alignment is important for the research design and also addresses construct and content validity (Skilling & Stylianides, 2020). In addition, context and content validity are addressed when the vignette material that portrays the hypothetical situations is fairly reflected and the users of the vignette resonate with it (Skilling & Stylianides, in press). Figure 11.3 represents a process for improving alignment between the constructed vignette method and the underlying aims and focus of the research which include aligning the vignette content with the theoretical framework, constructs, research design and methodology.

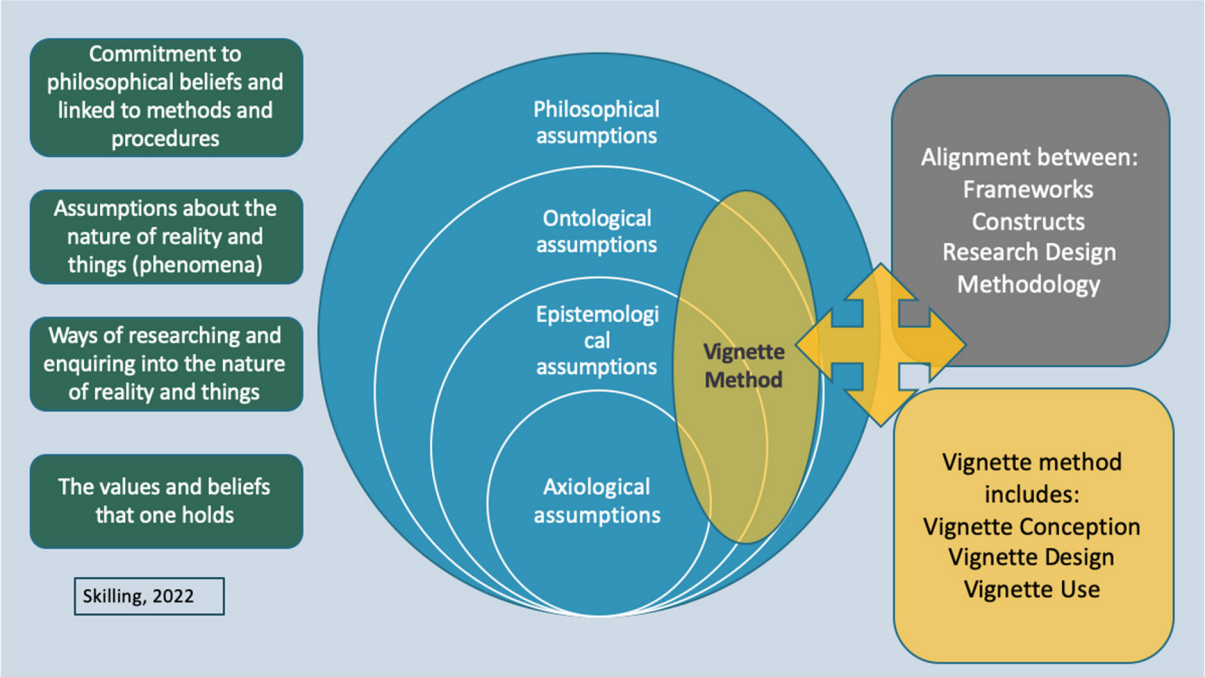


Figure 11.3 Methodological awareness and alignment with vignette construction elements

11.3 Introducing the Vignette framework

This framework was established to reflect the three main elements and sub-characteristics that are proposed as important aspects to consider when designing vignettes as the sole or main research methodological element (Table 11.1). The Vignette Framework attends to the vignettes' conception, design, and administration. On the right hand side of Table 11.1, descriptions of particular characteristics are provided (see Skilling & Stylianides, 2020, for a full discussion).

Table 1. Vignette framework: key elements, characteristics, and descriptors.

Key elements	Characteristics	Descriptors
Conception	Capturing content	Draw on conceptual or theoretical frameworks, existing literature, and practical experiences to reflect the essence of the research topic.
	Realistic and hypothetical portrayals	Portray characters and events that are representative of and meaningful to those experienced by the participants, balancing hypothetical yet realistic situations.
	Purpose/function	Construction guided by the research purpose, data sought, and respondents (e.g. promote/focus/stimulate discussion, solve problems, identify attitudes, seek beliefs, report practices, models of practice, norms, understandings). Vignette functions as the sole method or part of a multiphase data collection.
Design	Presentation	The nature of vignettes requires succinct (not necessarily complete or exact) portrayals of selected information. Brevity and incompleteness allow for participants to interpret/respond in unique and nuanced ways.
	Length	Written vignettes usually range between 50–200 words. Visual tools may be single or multiple images (e.g. comics). Video vignettes are typically a few minutes long. The length should consider maintaining interest, time for absorbing information and responding to it.
	Settings and terminology	Consider participants' degree of familiarity with the vignette situation (settings/ language specific to a particular cohort or profession) and ability to adequately respond to it. Also consider the appropriateness of using age-relevant and gender-neutral language.
	Open or closed questioning	Consider the purpose of the vignette to decide the type and format of questions. Open questions allow for more detailed, realistic, and independent reactions to the situation posed in vignettes. Questions may be in a written or verbal form (e.g. if vignettes are part of an interview situation).
	Participant perspectives	Consider from which perspective(s) the participants is (are) being asked to respond to the vignette (e.g. from a vignettes character's perspective, another role, or from their own perspective).
	Piloting	Pilot the vignette prior to use to assess the extent of how representative it is of situations and participants.
Administration	Instructions	Provide clear instructions for delivering, and how to respond to, the vignette.
	Timing and responses	Consider the phase within the research study the vignette will be given (e.g. as the staring point or to follow other data collection methods) and provide adequate time for responses.
	Delivery mode and frequency	Consider how the vignette will be delivered (e.g. in person, on-line) and how this might influence completion and quality of responses. Oral delivery may be appropriate but consider possible bias if read by the researcher or another. Multiple and frequent use may lead to a lack of responses and risk 'carry over' effects.

Skilling & Stylianides, 2020

Table 11.1 The Vignette Framework (Skilling & Stylianides, 2020, p. 4)

11.4 An example: aligning the research aims and vignette construction

This section provides a specific example of how two worded vignettes and accompanying questions were constructed and used in a research study with 40 in-service teachers. The overall aim of the study was to investigate the teachers' beliefs about promoting cognitive engagement (defined as self-regulation strategies use and metacognitive process) in early secondary mathematics classes.

The two written vignettes in Figure 11.4 reflect a test preparation situation by Teacher A and Teacher B. The construction of the vignettes adhered to the three main elements of the Vignette Framework which attended to its conception, design, and administration (Skilling & Stylianides, 2020). More specifically, the 'conception' element meant that each vignette drew on literature from the main constructs of cognitive engagement, self-regulation, metacognition, and related engagement to embed particular phrases as 'markers' which emphasised the different strategies and processes used by the two fictitious teachers (this provides the alignment

between the underlying theoretical frameworks and constructs). The two vignettes were deliberately worded to address similar aspects of cognitive engagement such as setting goals, planning, strategy choice, reflection and monitoring learning but to different degrees because the research was interested in the in-service teachers' beliefs about each situation (A and B) and to which teacher (A or B) they identified with. The 'design' element paid attention to the length of the vignettes, the appropriateness of the language used, and the context of the setting so that it resonated with those being asked to respond to it. The design element also considered the perspective respondents were expected to take (in this case their own) and the series of eight open-ended questions were connected to the phrases embedded in the written vignette narrative, represented in Figure 11.5. In a study where 40 in-service teachers responded to the written vignettes, all but one teacher identified the two teacher types in a similar way. This assured us that the context was valid as the situation portrayed in the vignettes resonated with practising teachers in English secondary schools. Further, their responses to the open-ended questions (and follow up interviews) yielded information indicative of professional reflection about the extent to which, and how, they promoted their students to self-regulate, monitor and reflect on their learning.

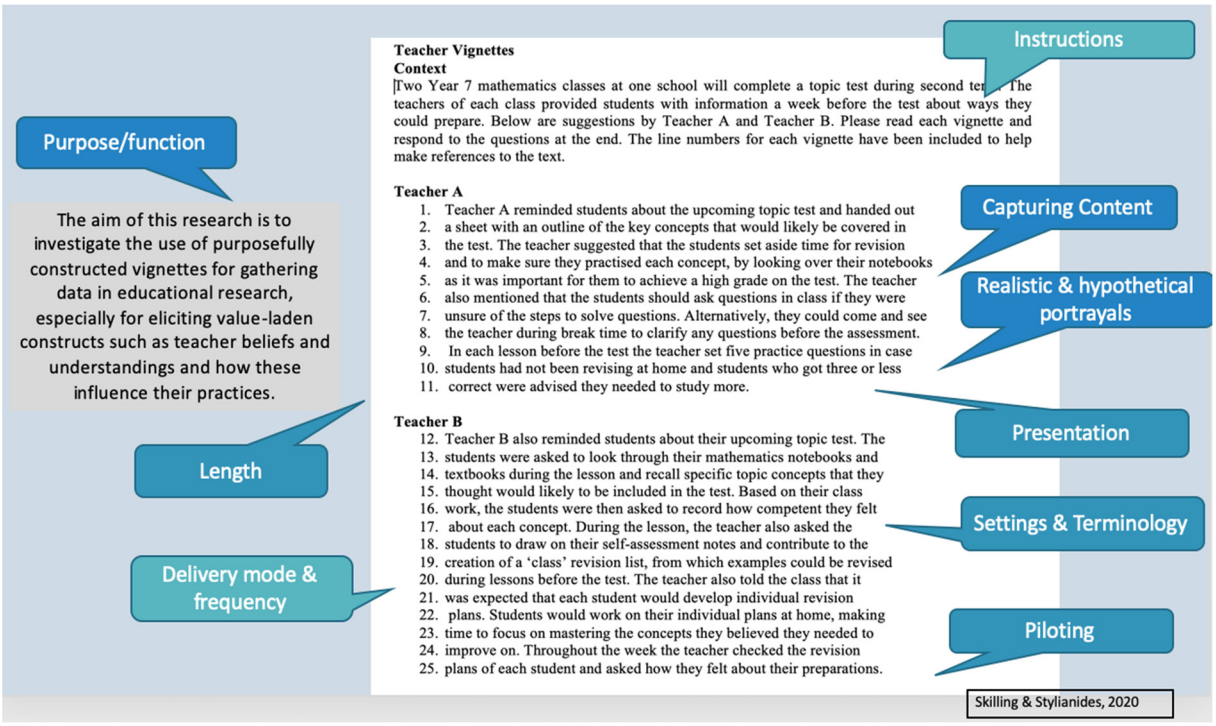


Figure 11.4: Teacher A and B cognitive engagement vignettes (guided by the Vignette Framework) (Skilling & Stylianides, 2020, p. 16)

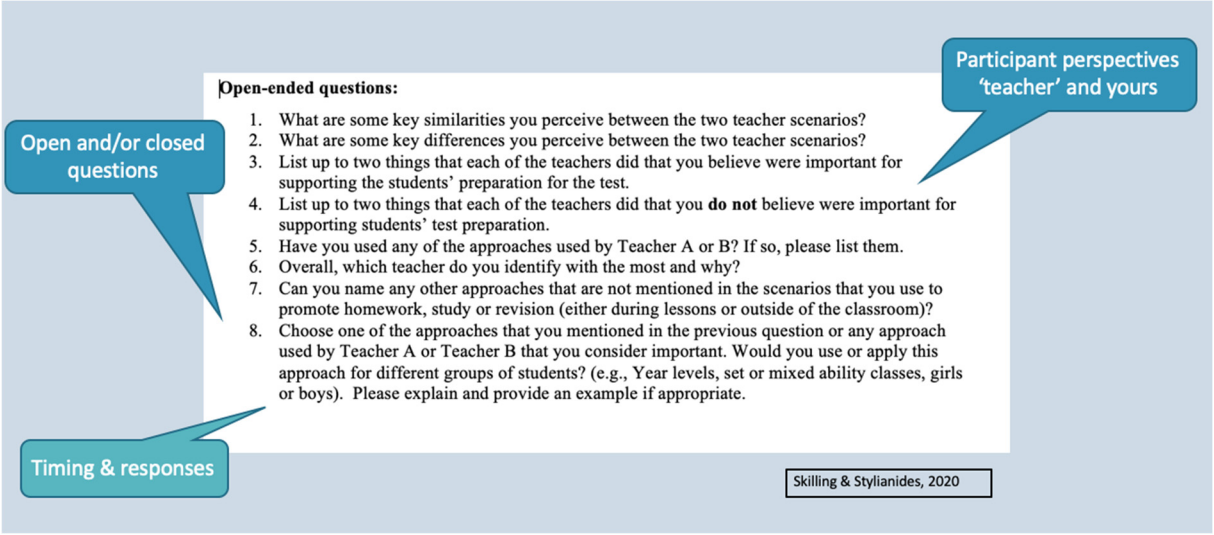


Figure 10.5: Teacher A and B cognitive engagement vignettes questions (guided by the *Vignette Framework*) (Skilling & Stylianides, 2020, p. 16)

It is argued that the attention paid to constructing the vignette by adhering to the elements and characteristics of the *Vignette Framework* which advocates for aligning key theoretical framing and construct with the phenomena of the vignette at the 'conception' phase is crucial for aligning the method with the research aims and in the process assist with establishing overall construct validity (Skilling & Stylianides, 2020). In addition, context and content validity are addressed when the vignette material that portrays the hypothetical situations is fairly reflected and the users of the vignette resonate with it (Skilling & Stylianides, in press).

11.5 Conclusion: Vignette validity in professional learning and research settings

In the deepening sections above, vignette validity was foregrounded as a requirement for research contexts, and how this requirement can be fulfilled through the Vignette Framework. However, also in professional learning opportunities, the use of vignettes should be reflected with respect of validity. The aspects of *practice validity*, *goal validity* and *theory validity* introduced in section 11.1 (see Figure 11.1) can be used to structure the process of reflecting on the validity of vignettes in professional learning contexts. Moreover, considerations on vignette validity in research can inform also vignette validity in contexts of professional learning.

12. Synthesis: Developing a coherent and adapted vignette-based approach with digital support

This activity leads the previous sections to a synthesis. Now, combining the possibilities of digitally created and/or represented vignettes come into play.

In learning support environments such as Moodle, text, cartoon, or video vignettes can be implemented easily, once they have been created as files. Questions related to these vignettes can be added and results of pre-service or in-service teachers' analysis can be collected online.

The digital support offered by the DIVER CREATE tool introduced in section 6 above offers further possibilities, as far as creating cartoon vignettes is concerned, in particular. The use of the DIVER tool can be combined with vignette-based working formats in larger course contexts, in line with their learning goals and the corresponding theoretical constructs.

You can base your considerations on your thoughts in the previous activities - Think of learning goals and constructs associated with a practice-relevant theoretical content domain: How could vignette-based learning activities be integrated in a digitally supported course framework? How can digital support facilitate content-centred and goal-oriented exchange among the participants? Can the DIVER Create tool bring an added value to the network of vignette-based learning opportunities?

Can you think of a framework of evaluating the participants' learning progress empirically? Are there possibilities of using vignettes in the evaluation instrument? How are learning goals and theoretical constructs implemented in corresponding vignettes? Can the instrument(s) be supported through (a) digital tool(s)? How could potential results from the evaluation feed back into the further course development?

13. Conclusions and outlook

The content of this booklet is not meant to be a completed collection of aspects related to vignette-based work in pre-service mathematics teacher education and in-service teacher professional development. Rather, it provides an overview of the potential of vignettes in terms of their formats and ways they can be used to support pre-service and in-service teachers as they train and develop their teacher expertise. Important, vignette-based professional-related learning opportunities are a dynamically developing field both as far as profession-related teaching interventions for mathematics teachers in various settings as well as for empirical research into aspects of mathematics teacher expertise (Buchbinder & Kuntze, 2018). Consequently, the activities included in this booklet play a key role: Depending on the reader's interest, the reader's approach and background, and last but not least the reader's work with the stimuli from the activities, this booklet is intended to yield individual progress for each reader's needs and interests in vignette-based work.

Vignettes have the potential of exemplary connections between mathematics education theory and practice contexts, such as classroom situations. We anticipate that this book provides a basis of understanding not only how vignettes can be developed and created to be effective as stimulus for teacher training and professional development but also for providing information about the credibility of vignettes as a method for educational research purposes. In addition, digital support tools for vignette-based work, particularly work with cartoon vignettes, can help to open up a variety of different ways of creating vignette-based learning opportunities related to the profession of mathematics teachers.

The project coReflect@maths is focusing these potentials. On www.coreflect.eu, more information about the project work and the project results is available. Updates on current further developments will be accessible on this webpage as well.

14. Acknowledgements

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This book aims to invite mathematics teacher educators to develop vignette-based professional learning settings for pre-service and in-service mathematics teachers. The stimuli provided in the book are designed so as to be adaptable to the readers' priorities in the field, as far as individual learning goals and formats for professional learning are concerned, whether empirical research e.g. for the evaluation of the teachers' learning should play a significant role or whether specific digitally-based representations of classroom situations are in the foreground, e.g. in view of potentials for distance learning.

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